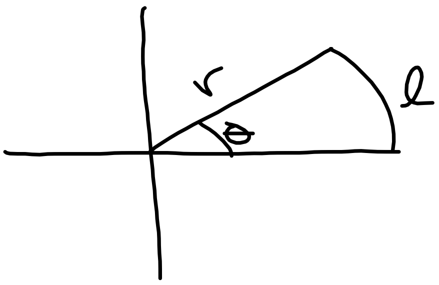
Unit 4: Rotational Motion and Torque

Angular Quantities

The Useful Way of Measuring Angles: The Radian



$\theta = \frac{\ell}{r}$

If $\theta = 360°$

$\ell = 2\pi r$

$\theta = \frac{2\pi \cancel{r}}{\cancel{r}} = 2\pi$

$1 \text{ rad} = \frac{360°}{2\pi} \approx 57.1°$

Angular Velocity and Acceleration

Linearly...

some position $\vec{x}(t)$

$$\vec{v}(t) = \frac{d}{dt}\vec{x}(t) = \frac{d\vec{x}(t)}{dt}$$

slope of position-time graph

$$\vec{a}(t) = \frac{d}{dt}\vec{v}(t) = \frac{d^2}{dt^2}\vec{x}(t)$$

slope of $\vec{v}$-t graph

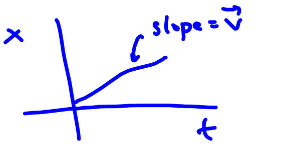
2nd derivative of $\vec{x}$-t graph (slope of the slope)

Angularly

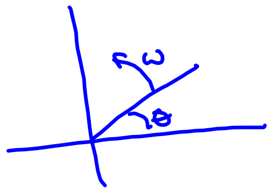
$$\vec{\omega}(t) = \frac{d\vec{\theta}(t)}{dt}$$

angular velocity

the rate of change of θ with time



$$\vec{x}_f = \vec{x}_i + \vec{v}t$$



If ω is constant

$$\theta_f = \theta_i + \omega t$$

$$\theta = \frac{l}{r}$$

$$\theta = \frac{l}{r}$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\frac{l_f - l_i}{r}}{\Delta t} = \frac{\Delta l}{r\Delta t} = \frac{v}{r}$$

speed.

$$\omega = \frac{v}{r}$$

$$\alpha = \frac{a}{r}$$

angular acceleration

Linear Equation

$v_f = v_i + at$

$d = v_i t + \frac{1}{2}at^2$

$d = v_f t - \frac{1}{2}at^2$

$v_f^2 = v_i^2 + 2ad$

$d = \left(\frac{v_i + v_f}{2}\right)t$

a is constant

$\Sigma\vec{F} = m\vec{a}$

Angular Equation

$\omega_f = \omega_i + \alpha t$

$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$\theta = \omega_f t - \frac{1}{2}\alpha t^2$

$\omega_f^2 = \omega_i^2 + 2\theta\alpha$

$\theta = \left(\frac{\omega_i + \omega_f}{2}\right)t$

α is constant

$\Sigma\vec{\tau} = I\vec{\alpha}$

Relating Angular Velocity to Linear Velocity

Exercise:

Show $a = r\alpha$