Momentum

Definition: The product of an object's mass and velocity.

$$\vec{p} = m\vec{v}$$

Newton's 2nd Law

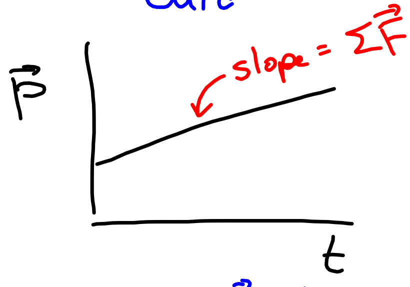
$$\Sigma\vec{F} = m\vec{a}$$

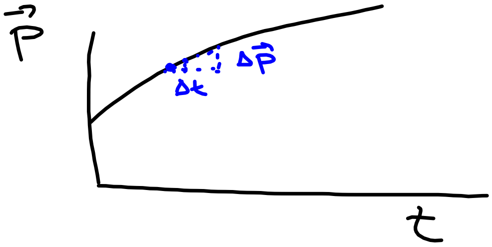
nope.

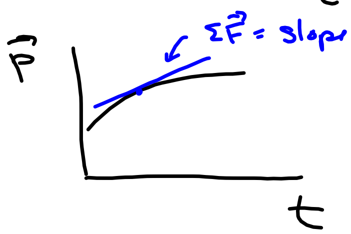
$$\Sigma\vec{F} = \frac{d\vec{p}}{dt}$$

derivative (slope)

what Newton actually said







$$\frac{d\vec{p}}{dt} = \lim_{t \to 0} \frac{\Delta\vec{p}}{\Delta t}$$

Non calculus form

$$\vec{v}_{ave} = \frac{\Delta\vec{d}}{\Delta t}$$

$$\Sigma\vec{F}_{ave} = \frac{\Delta\vec{p}}{\Delta t}$$

$$= \frac{\Delta(m\vec{v})}{\Delta t}$$

$$= \frac{m_f\vec{v}_f - m_i\vec{v}_i}{\Delta t}$$

If $m_i = m_f = m$

$$= \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t}$$

$$\Sigma\vec{F}_{ave} = m\vec{a}_{ave}$$ is the case of Newton's 2nd Law in which m is constant

This also applies at any instant in time.

Momentum-Impulse Theorem

cause — effect.

$$\vec{J} = \Sigma \vec{F}_{avg} \Delta t = \Delta \vec{p}$$

impulse

definition

the product of a net force acting on an object and the period of time over which it acts.

$$\Sigma \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

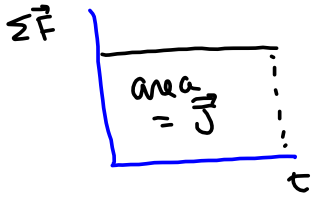
$$\Sigma \vec{F} \Delta t = \Delta \vec{p}$$

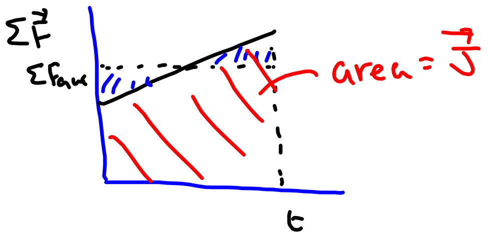
Calculus version

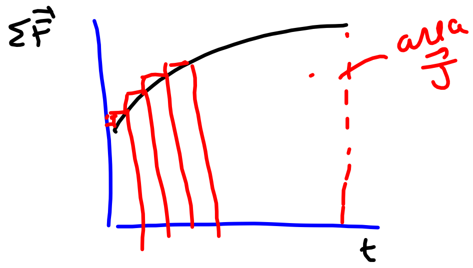
$$J = \int_{t_1}^{t_2} \Sigma \vec{F}(t)\,dt = \int_{t_1}^{t_2} d\vec{p}$$

find area between t_1 and t_2 under $\Sigma \vec{F}(t)$

integral (area)

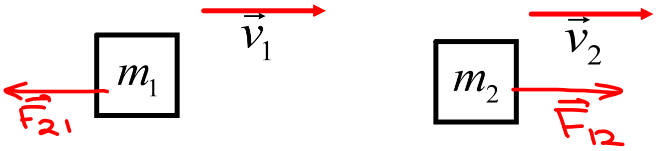






Conservation of Momentum

Consider two masses, m_1 traveling with some velocity, $\vec{v}_1$, and m_2 traveling with some velocity, $\vec{v}_2$, and they collide.



$F_{12} = F_{21}$, $\vec{F}_{12} = -\vec{F}_{21}$ } Newton's 3rd law.

After the collision, the masses have velocities of $\vec{v}_1'$ and $\vec{v}_2'$ repsectively.

(Note: This is a general assumption - it could be linear, the masses could stick together, or it could be multi-dimensional. For a complete proof, we would have to assume N masses and the interactions between, but it follows from this)

By momentum-impulse theorem (Newton's 2nd law)

$$\Delta\vec{p}_1 = \sum\vec{F}_1 \Delta t_1 \quad \text{and} \quad \Delta\vec{p}_2 = \sum\vec{F}_2 \Delta t_2$$

$$\Delta\vec{p}_1 = \vec{F}_{21} \Delta t_1 \qquad \Delta\vec{p}_2 = \vec{F}_{12} \Delta t_2$$

but $\Delta t_1 = \Delta t_2 = \Delta t$ (Newton's 3rd)

So $\Delta\vec{p}_1 = \vec{F}_{21} \Delta t \qquad \Delta\vec{p}_2 = -\vec{F}_{21} \Delta t$

So $\Delta\vec{p}_1 = -\Delta\vec{p}_2$

$$m_1\vec{v}_1' - m_1\vec{v}_1 = -(m_2\vec{v}_2' - m_2\vec{v}_2)$$

$$\vec{p}_1' - \vec{p}_1 = -(\vec{p}_2' - \vec{p}_2)$$

$$\vec{p}_1' + \vec{p}_2' = \vec{p}_1 + \vec{p}_2$$

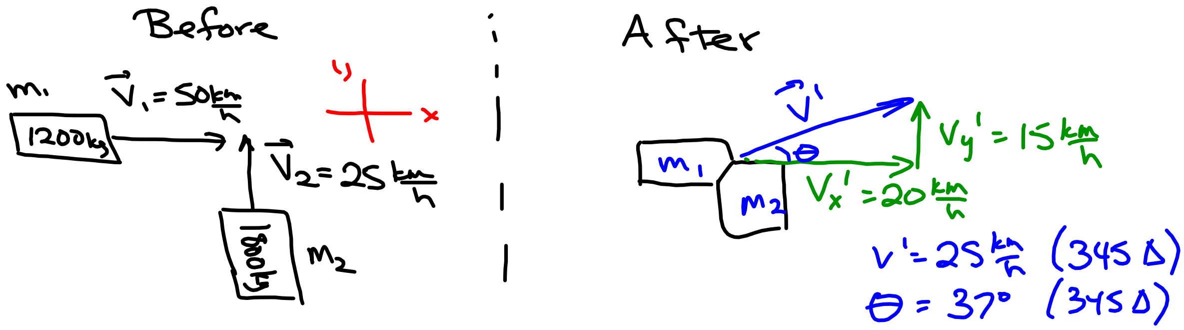
So $\sum\vec{p}\,' = \sum\vec{p}$

Law of cons. of momentum.

True in a closed system (no net external forces)

Example 1:

A car (m=1200 kg) travelling East with a speed of 50 km/h collides with a truck (m=1800 kg) travelling North with a speed of 25 km/h. They stick together after the collision. What is the resulting velocity of the vehicles *immediately after* they collide?



$\Sigma\vec{p} = \Sigma\vec{p}\,'$ ⇒ true in both x and y (E-W) (N-S)

$$\Sigma p_x = \Sigma p_x'$$

$$m_1 V_1 = (m_1 + m_2) V_x'$$

$$1200\,kg(+50\tfrac{km}{h}) = (1200 + 1800) V_x'$$

$$V_x' = +20\tfrac{km}{h} \quad (20\tfrac{km}{h}\ E)$$

$$\Sigma p_y = \Sigma p_y'$$

$$m_2 V_2 = (m_1 + m_2) V_y'$$

$$1800\,kg(+25\tfrac{km}{h}) = (1200 + 1800\,kg) V_y'$$

$$V_y' = +15\tfrac{km}{h} \quad (15\tfrac{km}{h}\ N)$$

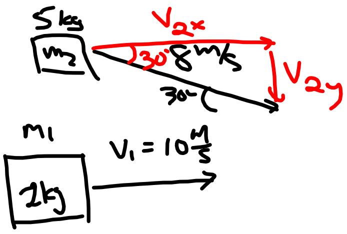
$$\vec{V}' = 25\tfrac{km}{h}\ E37°N$$

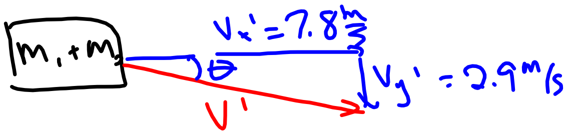
(053°)

(N 53° E)

Example 2:

A mass of 2 kg travelling to the right with a speed of 10 m/s collides with a mass of 5 kg travelling at a speed of 8 m/s at an angle of 30^0 to the right. They stick together after the collision. What is the resulting velocity of the masses *immediately after* they collide?





$$\Sigma p_x = \Sigma p_x'$$
$$m_1 V_1 + m_2 V_{2x} = (m_1 + m_2) V_x'$$
$$2(10) + 5(8\cos 30^\circ) = (2+5) V_x'$$
$$54.6\ \text{kg}\cdot\frac{m}{s} = 7\text{kg}\ V_x$$
$$V_x' = 7.8\frac{m}{s}$$

$$\Sigma p_y = \Sigma p_y'$$
$$m_2 V_{2y} = (m_1 + m_2) V_y'$$
$$5(8\tfrac{m}{s}\sin 30^\circ) = (2+5) V_y'$$
$$20\ \text{kg}\cdot\frac{m}{s} = 7\text{kg}\ V_y'$$
$$V_y' = 2.9\ m/s$$

$$\vec{V}' = 8.4\frac{m}{s}\ \ 20^\circ \text{ to right.}$$

$$c^2 = a^2 + b^2$$
$$V'^2 = (2.9\tfrac{m}{s})^2 + (7.8\tfrac{m}{s})^2$$
$$V' = 8.4\ m/s$$
$$\tan\theta = \frac{2.9\frac{m}{s}}{7.8\frac{m}{s}}$$
$$\theta = 20^\circ$$

Homework:
p 509 # 35-37
Read pp. 503 - 508