## Momentum

Definition: The product of an object's mass and velocity.

$\vec{p} = m\vec{v}$

Newton's 2nd Law

$\sum \vec{F} = m\vec{a}$

nope.

Actually said

$\sum \vec{F} = \frac{d\vec{p}}{dt}$

derivative (slope)

Non-calculus version

$\vec{v}_{ave} = \frac{\Delta \vec{d}}{\Delta t}$

$\sum \vec{F}_{ave} = \frac{\Delta \vec{p}}{\Delta t}$

If $m_f = m_i = m$

$= \frac{m_f \vec{v}_f - m_i \vec{v}_i}{\Delta t}$

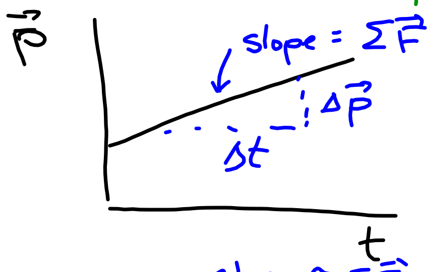
$= \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t}$

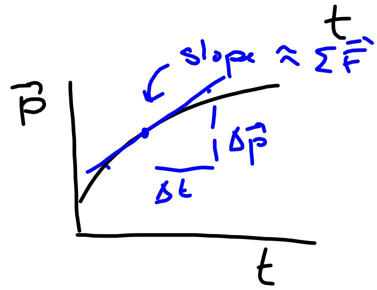
$= m \frac{\Delta \vec{v}}{\Delta t}$

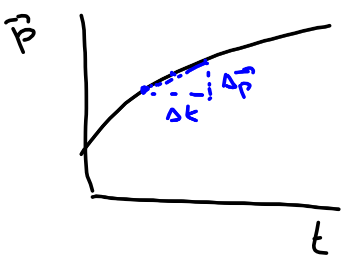
$= m\vec{a}$

The special case of Newton's 2nd Law where $m, \vec{a}$ are constant.

This applies also at ANY moment in time.




$\frac{d\vec{p}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t}$

Momentum-Impulse Theorem

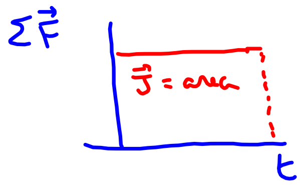
Non calculus version

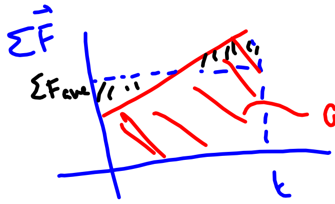
$$\underbrace{\vec{J} = \overbrace{\Sigma \vec{F}_{ave} \Delta t}^{\text{cause}}}_{\text{definition}} = \overbrace{\Delta \vec{p}}^{\text{effect}}$$

$$\Sigma \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

$$\Sigma \vec{F} \Delta t = \Delta \vec{p}$$

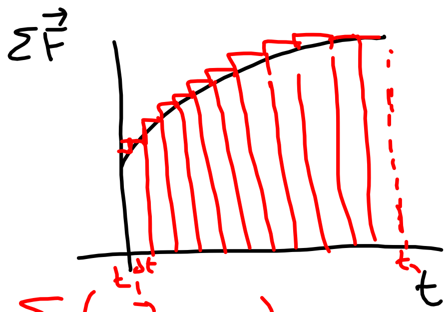
$$\vec{J} = \int_{t_1}^{t_2} \Sigma \vec{F}(t)\, dt = \int_{t_1}^{t_2} d\vec{p}$$




$$\Sigma \vec{F}_{ave} \Delta t = \vec{J}$$

area = $\vec{J}$

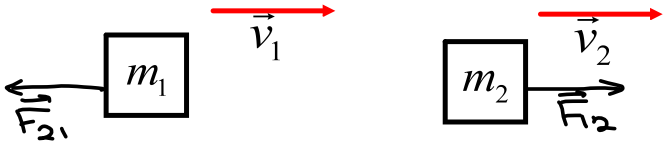



$$\vec{J} \approx \sum \left( \Sigma \vec{F}(t) \Delta t \right)$$

$$\vec{J} = \int_{t_1}^{t_2} \Sigma \vec{F}(t)\, dt$$

## Conservation of Momentum

Consider two masses, $m_1$ traveling with some velocity, $\vec{v}_1$ , and $m_2$ traveling with some velocity, $\vec{v}_2$ , and they collide.




$$F_{12} = F_{21}$$
$$\vec{F}_{12} = -\vec{F}_{21}$$

} Newton's 3rd

After the collision, the masses have velocities of $\vec{v}_1'$ and $\vec{v}_2'$ repsectively.

(Note: This is a general assumption - it could be linear, the masses could stick together, or it could be multi-dimensional. For a complete proof, we would have to assume N masses and the interactions between, but it follows from this)

Newton's 2nd law

$$\Delta\vec{p}_1 = \sum\vec{F}_1 \Delta t_1 \qquad \Delta\vec{p}_2 = \sum\vec{F}_2 \Delta t_2$$
$$= \vec{F}_{21} \Delta t_1 \qquad = \vec{F}_{12} \Delta t_2$$

But $\Delta t_1 = \Delta t_2 = \Delta t$ (Newton's 3rd)

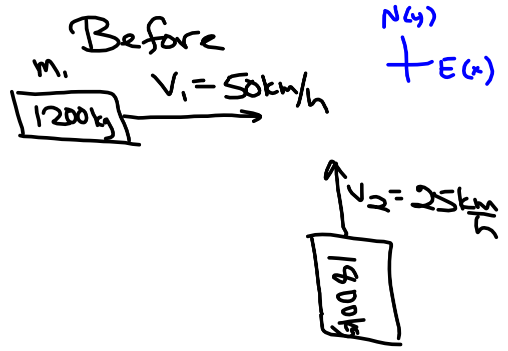
$$\Delta\vec{p}_1 = \vec{F}_{21} \Delta t \qquad \Delta\vec{p}_2 = -\vec{F}_{21} \Delta t$$
$$\Delta\vec{p}_1 = -\Delta\vec{p}_2$$
$$\vec{p}_1' - \vec{p}_1 = -(\vec{p}_2' - \vec{p}_2)$$
$$\vec{p}_1' + \vec{p}_2' = \vec{p}_1 + \vec{p}_2$$
$$\sum\vec{p}' = \sum\vec{p}$$

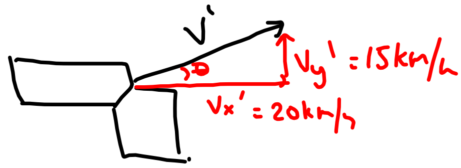
Law of cons. of momentum

True in closed systems.
- no mass in or out
- no external net forces to the system.

Example 1:

A car (m=1200 kg) travelling East with a speed of 50 km/h collides with a truck (m=1800 kg) travelling North with a speed of 25 km/h. They stick together after the collision. What is the resulting velocity of the vehicles *immediately after* they collide?




$V' = 25$ km/h (3-4-5 △)

$\theta = 37°$ (3-4-5 △)

$\vec{V}' = 25$ km/h E37°N

N53°E

(053°)

$$\Sigma \vec{p} = \Sigma \vec{p}'$$

$$\Sigma \vec{p}_x = \Sigma \vec{p}_x'$$

$$m_1 v_1 = (m_1 + m_2) V_x'$$

$$(1200\text{kg})(+50\tfrac{\text{km}}{\text{h}}) = (1200 + 1800\text{kg}) V_x'$$

$$V_x' = +20 \text{ km/h}$$

(20 km/h E)

$$\Sigma \vec{p}_y = \Sigma \vec{p}_y'$$

$$m_2 V_2 = (m_1 + m_2) V_y'$$

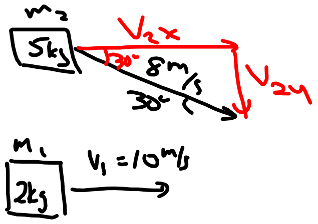
$$(1800\text{kg})(25\text{ km/h}) = (1200 + 1800\text{kg}) V_y'$$

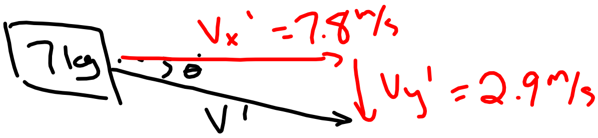
$$V_y' = +15 \text{ km/h}$$

(15 km/h N)

## Example 2:

A mass of 2 kg travelling to the right with a speed of 10 m/s collides with a mass of 5 kg travelling at a speed of 8 m/s at an angle of $30^0$ to the right. They stick together after the collision. What is the resulting velocity of the masses *immediately after* they collide?




$\Sigma \vec{p}_x = \Sigma \vec{p}_x{}'$

$m_1 v_1 + m_2 v_{2x} = (m_1 + m_2) v_x'$

$2(10) + 5(8\cos 30°) = 7 v_x'$

$54.6 = 7 v_x'$

$v_x' = 7.8 \frac{m}{s}$ right

$\Sigma \vec{p}_y = \Sigma \vec{p}_y{}'$

$m_2 v_{2y} = (m_1 + m_2) v_y'$

$5(8\sin 30°) = 7 v_y'$

$20 = 7 v_y'$

$v_y' = 2.9\, m/s$ down

$c^2 = a^2 + b^2$

$v'^2 = (7.8)^2 + (2.9)^2$

$v' = 8.4\, m/s$

$\tan\theta = \frac{2.9}{7.8}$

$\theta = 20°$

$\vec{v}\,' = 8.4\, m/s$ 20° to right.

Homework:
p 509 # 35-37
Read pp. 503 - 508