

Simple Harmonic Motion

Definitions:

Periodic Motion - a motion that repeats itself over regular intervals of time.



Period (T) - the time to complete one cycle of the motion measured in s.

Frequency (f) - the number of cycles per unit time
(sometimes ν)
↑
nu.

$$f = \frac{1}{T}$$

unit $[s^{-1}] = [Hz]$

Springs

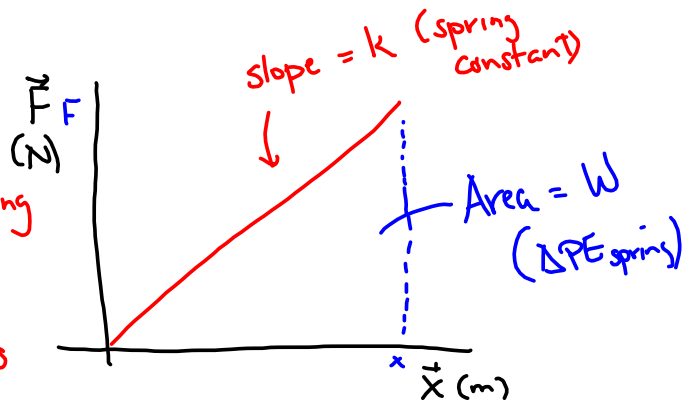
reaction forces

$$\vec{F} = k \vec{x}$$

← force to stretch spring

$$\vec{F} = -k \vec{x}$$

← force the spring applies
for SHM.



$$\text{Area} = \frac{bh}{2}$$

$$= \frac{1}{2} x \cdot F \quad \text{but } F = kx$$

$$= \frac{1}{2} x \cdot kx$$

$$PE_s = \frac{1}{2} kx^2$$

$$PE = \odot \text{ when } x = \ominus \text{ (defined)}$$

Amplitude - maximum change from equilibrium
(A)

Equilibrium - $\Sigma F = 0$, $\Sigma \tau = 0$

The position where $\Sigma F = 0$ and $\Sigma \tau = 0$

3 types:

Stable



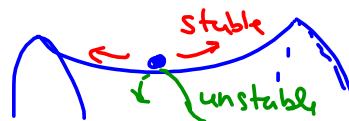
If we displace the object by a small Δx , a force moves it back to equilibrium

Unstable



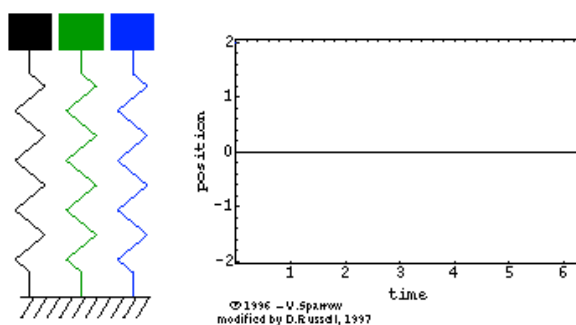
Small Δx , moves away from eq.

Saddle point



So what does this have to do with a circle, anyway?

http://www.phy.hk/wiki/j/Eng/springSHM/springSHM_js.htm



Velocity and Acceleration of a SHO (Simple harmonic oscillator)

A SHO has a restorative force of the form

$$\vec{F} = -k\vec{x} \quad (\text{like a spring, or the shadow of circular motion})$$

For a spring

$$\underbrace{\frac{1}{2}kA^2}_{PE_{max}} = \underbrace{\frac{1}{2}mv^2}_{KE \text{ at some point } x} + \underbrace{\frac{1}{2}kx^2}_{PE \text{ at some point } x}$$

$$k(A^2 - x^2) = mv^2$$

$$v = \sqrt{\frac{kA^2}{m} \left(1 - \left(\frac{x}{A}\right)^2\right)}$$

$$= \sqrt{\frac{kA^2}{m}} \sqrt{1 - \left(\frac{x}{A}\right)^2}$$

Speed at some point x →

$$v(x) = \sqrt{\frac{k}{m}} A \sqrt{1 - \left(\frac{x}{A}\right)^2}$$

if $x = 0$ $v(0) = v_{max} = v_0$
 $v_0 = \underline{\underline{\sqrt{\frac{k}{m}} A}}$

$$\vec{F} = -k\vec{x} = m\vec{a} \quad \vec{a}(x) = -\frac{k}{m}\vec{x}$$

$$\vec{v}(x) = \pm v_0 \sqrt{1 - \left(\frac{x}{A}\right)^2}$$

Homework Read pp. 600-608,
Problems p. 608 #1-4