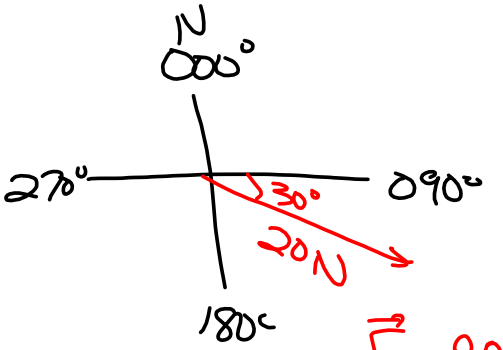
Vector Notation

$1 + 1 = 2$

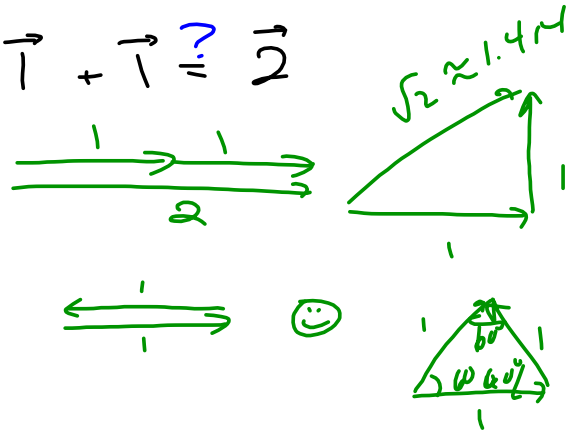


$\vec{F}$: 20N 120°

E30°S

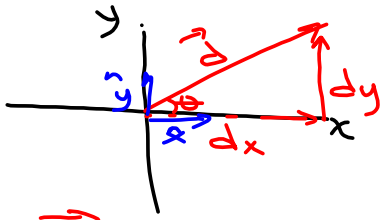
S60°E

$\vec{1} + \vec{1} \stackrel{?}{=} \vec{2}$



Unit Vectors

2-D Cartesian



$d = |\vec{d}| = \|\vec{d}\|$
$d_x = |\vec{d}_x|$

↑ norm (length) of a vector

$d_x = d\cos\theta$
$d_y = d\sin\theta$

$\vec{d}_x + \vec{d}_y = \vec{d}$

$d = \sqrt{d_x^2 + d_y^2}$

$\vec{d} = \underbrace{d_x\hat{x}}_{\vec{d}_x} + \underbrace{d_y\hat{y}}_{\vec{d}_y}$

Define $\hat{n}$ (n-hat) to be a vector of length 1 in the increasing (+) n-direction

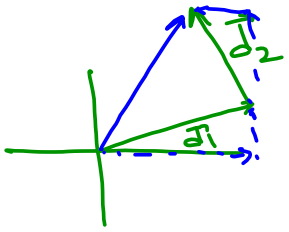
$\hat{x}$ → vector of length 1 in +X-dir.

Addition

$\vec{d}_1 = d_{1x}\hat{x} + d_{1y}\hat{y}$

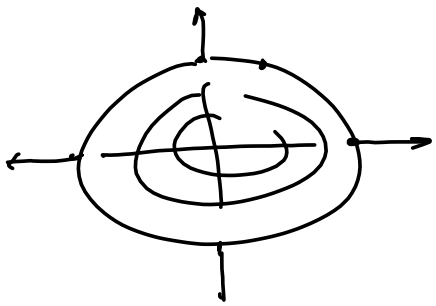
$+ \quad \vec{d}_2 = d_{2x}\hat{x} + d_{2y}\hat{y}$

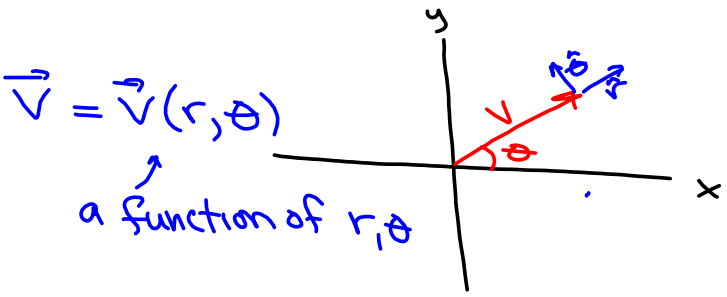
$\vec{d}_1 + \vec{d}_2 = (d_{1x}+d_{2x})\hat{x} + (d_{1y}+d_{2y})\hat{y}$



Polar Coordinates

$\vec{V} = V\hat{r}$





$\vec{V} = \vec{V}(r,\theta)$

↑ a function of r,θ

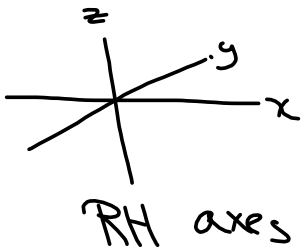
$\hat{r} = \hat{r}(\theta)$: radially outward

$\hat{\theta} = \hat{\theta}(\theta)$: CCW

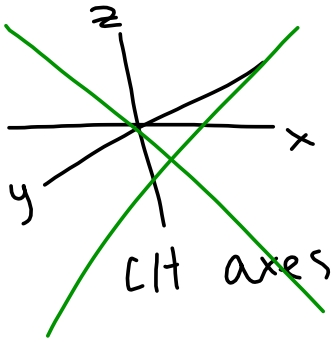
$\vec{V} = -5\frac{m}{s}\hat{\theta}$ is a speed of $5\frac{m}{s}$ in the CW dir.

3D

Cartesian

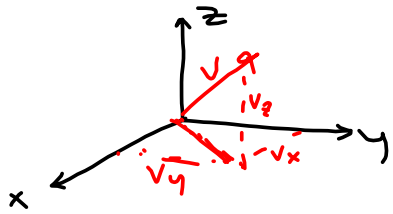


RH axes



LH axes

BAD

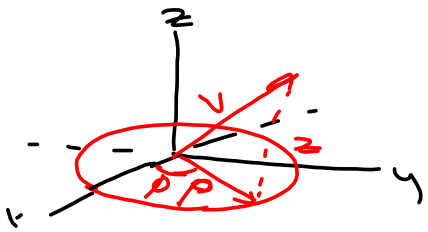


$\vec{V} = \vec{V}(x, y, z)$

$= V_x \hat{x} + V_y \hat{y} + V_z \hat{z}$

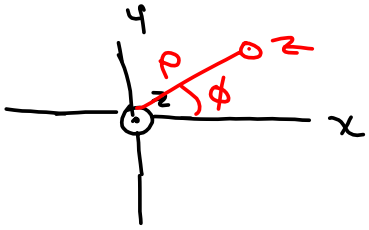
$V = \sqrt{V_x^2 + V_y^2 + V_z^2}$

Cylindrical Coordinates



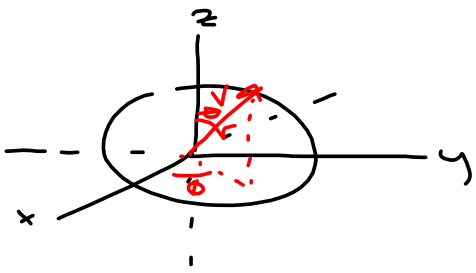
polar in x-y plane
+ a z-axis

$\vec{V} = V(\rho, \phi, z)$



⊙ out of paper
⊗ into paper

Spherical



$\vec{V} = \vec{V}(r, \theta, \phi)$

$0^\circ \leq \theta \leq 180^\circ$

$0^\circ \leq \phi \leq 360^\circ$

Vector "Multiplication"

$\vec{V} = V_x\hat{x} + V_y\hat{y} + V_z\hat{z}$

$\vec{W} = W_x\hat{x} + W_y\hat{y} + W_z\hat{z}$

$\vec{V}\vec{W}$ = DNE

2 types of vector products

① Dot product (scalar) $\vec{V}\cdot\vec{W}$

② Cross product (vector) $\vec{V}\times\vec{W}$

$\vec{V}\cdot\vec{W} = V_xW_x + V_yW_y + V_zW_z$

$\hat{x}\cdot\hat{x} = \hat{y}\cdot\hat{y} = \hat{z}\cdot\hat{z} = 1$

$\hat{x}\cdot\hat{y} = \hat{y}\cdot\hat{x} = \ldots = 0$

$\vec{V}\cdot\vec{W} = |\vec{V}||\vec{W}|\cos\theta$

θ: angle between $\vec{V}$ and $\vec{W}$

$W = F_{\parallel}d$

$= \vec{F}\cdot\vec{d}$