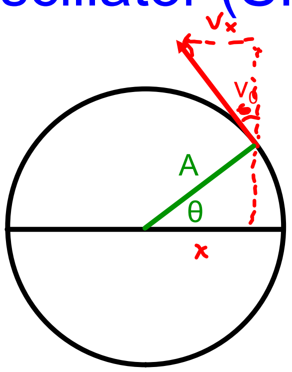
The Simple Harmonic Oscillator (SHO)

Consider an object traveling in a circle with speed v_0.
Now consider the x-component of the velocity.



$v_x = v_0 \sin\theta$

$\Rightarrow v_x = v_0 \sqrt{1-\left(\frac{x}{A}\right)^2}$

same a V(x) of spring

$\sin\theta = \frac{y}{A}$

$= \frac{\sqrt{A^2 - x^2}}{A}$

$= \sqrt{1-\left(\frac{x}{A}\right)^2}$

Recall for a spring $v_0 = \sqrt{\frac{k}{m}}\, A$

Since same x, $\frac{dx}{dt}$ as a spring for all x then $\frac{d^n x}{dt^n}$ is the same for all x.

$x \in [-A, A]$

x belongs to the set of -A to A inclusive

$-A \leq x \leq A$

Period

Period of circle $v_0 = \frac{2\pi A}{T}$

$$T = \frac{2\pi A}{v_0}$$

So for a spring $T = \frac{2\pi A}{v_0} = \frac{2\pi \cancel{A}}{\sqrt{\frac{k}{m}}\cancel{A}} = 2\pi\sqrt{\frac{m}{k}}$

Frequency

$f = \frac{1}{T}$ circle $f = \frac{v_0}{2\pi A}$

AP equation sheet.

spring $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Position

What about θ as a function of time?

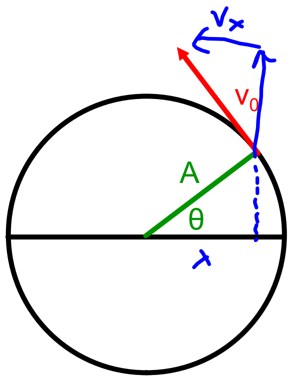
If $\theta_0 = 0$

$\theta = \omega t$

$= 2\pi f t$

$\omega = \frac{v_0}{A} = 2\pi f$

$= \frac{2\pi}{T}$



So $x = A\cos\theta = A\cos 2\pi f t$

(Note: if $\theta_0 \neq 0$ this becomes $A\cos(2\pi f t + \theta_0)$)

What does this mean for the velocity?

$$v = v_0\sqrt{1-\left(\frac{x}{A}\right)^2}$$

$$= v_0\sqrt{1-\left(\frac{\cancel{A}\cos 2\pi f t}{\cancel{A}}\right)^2}$$

$$= v_0\sqrt{1-\cos^2 2\pi f t}$$

$$= v_0\sqrt{\sin^2 2\pi f t}$$

$$= -v_0 \sin 2\pi f t$$

$\sin^2\theta + \cos^2\theta = 1$

$\forall\,\theta$

for all

If $0 < \theta \leq 90°$ v is to the left

So for a spring, $v_0 = \sqrt{\frac{k}{m}}\,A$

$$v(t) = -\sqrt{\frac{k}{m}}\,A\sin 2\pi f t$$

For a circle $v_0 = \frac{2\pi A}{T} = 2\pi f A$

$$v(t) = -2\pi f A \sin 2\pi f t$$ (which is ALSO true for a spring!)

Acceleration

Note: $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$\cos\sqrt{\frac{k}{m}}\,t$

For a spring

$$\vec{F} = -kx = ma$$

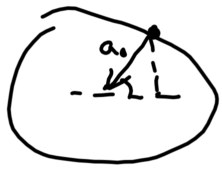
$$a = -\frac{k}{m}x = -\frac{k}{m}A\cos 2\pi f t$$

For a circle

$$a_x = -a_0\cos\theta$$

$$= -4\pi^2 f^2 A\cos 2\pi f t$$

$a_0 = \frac{v_0^2}{A}$



$v_0 = 2\pi f A$

$a_0 = 4\pi^2 f^2 A$

The whole thing again.... using calculus!

$$x(t) = A \cos 2\pi f t$$

$$v = \frac{dx}{dt} = \frac{d}{dt} A \cos 2\pi f t$$

$$= A \frac{d}{dt} \cos \underbrace{2\pi f t}$$

$$= A(-\sin 2\pi f t) \cdot \frac{d}{dt}(2\pi f t)$$

$$= -2\pi f A \sin 2\pi f t$$

$$\frac{d}{dt} \cos t = -\sin t$$

but

$$\frac{d}{dt} \cos Bt = -B \sin t$$

$$a = \frac{d^2x}{dt^2} = \frac{d}{dt}(-2\pi f A \sin 2\pi f t)$$

$$= -4\pi^2 f^2 A \cos 2\pi f t$$

Same as before.