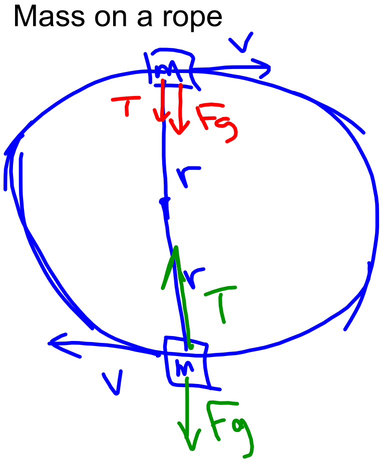
Vertical Circles

Mass on a rope



At top

$$\Sigma F = F_g + T = ma_c$$

$$mg + T = \frac{mv^2}{r}$$

$$T = \frac{mv^2}{r} - mg$$

$$= m\left(\frac{v^2}{r} - g\right)$$

At bottom

$$\Sigma F = T - mg = \frac{mv^2}{r}$$

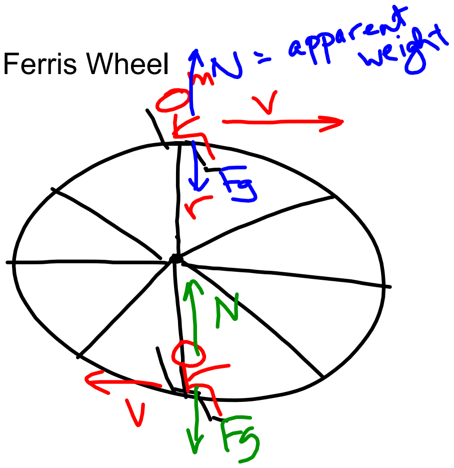
$$T = \frac{mv^2}{r} + mg$$

What is the minimum v that will still allow the mass to go in a circle? $\Rightarrow$ $T = 0$ so $\cancel{m}g = \frac{\cancel{m}v^2}{r}$

$$v = \sqrt{gr}$$

Maximum speed? ↳ dependent on breaking strength of rope (T_{max})

Ferris Wheel



<u>At top</u>

$$\Sigma F = F_g - N = \frac{mv^2}{r}$$

$$N = mg - \frac{mv^2}{r}$$

<u>At bottom</u>

$$\Sigma F = N - F_g = \frac{mv^2}{r}$$

$$N = mg + \frac{mv^2}{r}$$

V_{min} = ☺

$V_{max} = ?$

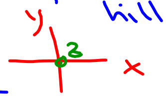
V_{max} when $N =$ ☺

$$mg = \frac{mv^2}{r}$$

$$v = \sqrt{gr}$$

Banked Turns

Assume it does not slide up or down hill



$\mu \neq 0$

Top

Lets assume V_{max} for turn

y-dir $\Sigma F = 0$

$N_y = F_g + f_y$ $\quad$ f_{max} if V_{max} or V_{min}

$N\cos\theta = mg + \mu N \sin\theta$

$N(\cos\theta - \mu \sin\theta) = mg$

$$N = \frac{mg}{\cos\theta - \mu\sin\theta}$$

x-dir $\Sigma F = ma = \frac{mv^2}{r}$

$\Sigma F = f_x + N_x = \frac{mv^2}{r}$

$\mu N\cos\theta + N\sin\theta = \frac{mv_{max}^2}{r}$

$N(\sin\theta + \mu\cos\theta) = \frac{mv_{max}^2}{r}$

$$\cancel{m}g\,\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{\cancel{m}v_{max}^2}{r}$$

$$V_{max} = \sqrt{\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\, gr}$$

For v_{min}, friction flips

$$V_{min} = \sqrt{\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\, gr}$$

$V_{min} \leq V \leq V_{max}$ to navigate the turn safely.

$V_{min} = 0$ when $\sin\theta = \mu\cos\theta$

$\mu = \tan\theta$

What if μ is negligible?

$V_{max} = V_{min} = \sqrt{gr\tan\theta}$

If θ is 0 $\quad$ V_{min} is not defined ($v_{min} = 0$)

$$V_{max} = \sqrt{\frac{\mu\cos\theta}{\cos\theta}\, gr}$$

$$= \sqrt{\mu g r}$$