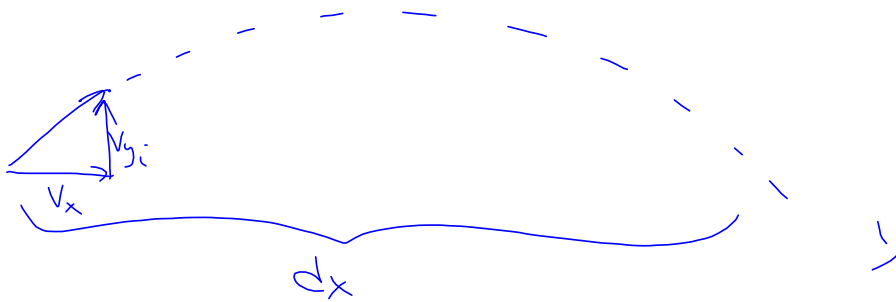


Symmetrical Projectiles Pt II



$$d_x = v_x t$$

$$d = \text{☺}$$

$$v_i = v_{y_i} \quad v_f = -v_{y_i}$$

$$a = -g$$

$$d = v_i t + \frac{1}{2} a t^2$$

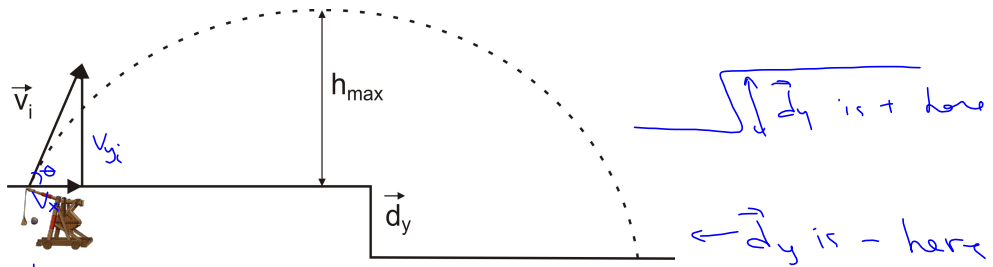
$$\text{☺} = v_{y_i} t - \frac{1}{2} g t^2$$

$$\text{☺} = t \left(v_{y_i} - \frac{1}{2} g t \right)$$

$$t = \text{☺} \text{ OR } v_{y_i} - \frac{1}{2} g t = \text{☺}$$

$$t = \frac{2v_{y_i}}{g}$$

Asymmetrical Projectiles



\vec{d}_y is + here

\vec{d}_y is - here

x
 $dx = v_x t$
 $= v_x (by\ mass)$

y
 Going up
 $v_i = v_{yi}$
 $a = -g$
 $v_f = \ominus$
 $v_f = v_i + at$
 $\ominus = v_{yi} - gt$
 $t_{up} = \frac{v_{yi}}{g}$

$v_f^2 = v_i^2 + 2ad$
 $\ominus = v_{yi}^2 - 2gh_{max}$
 $h_{max} = \frac{v_{yi}^2}{2g}$

y
 Coming down
 $v_i = \ominus$
 $a = -g$
 $d = -(h_{max} - d_y)$
 $d = v_i t + \frac{1}{2}at^2$
 $-(h_{max} - d_y) = -\frac{1}{2}gt^2$
 $t_{down} = \sqrt{\frac{2(h_{max} - d_y)}{g}}$

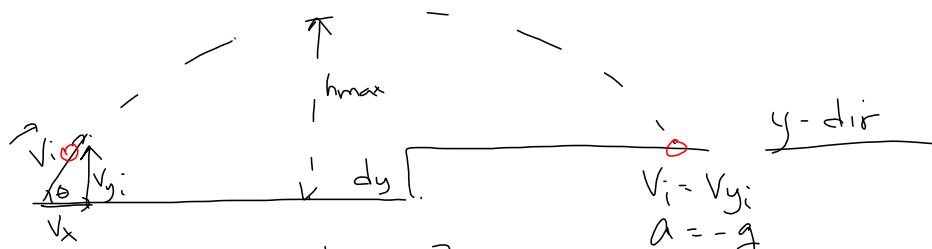
$v_f^2 = v_i^2 + 2ad$

$v_f = -\sqrt{2g(h_{max} - d_y)}$

$t_T = t_{up} + t_{down}$
 $= \frac{v_{yi}}{g} + \sqrt{\frac{2(h_{max} - d_y)}{g}}$

If $d_y = \ominus \Rightarrow$ symmetrical

$t = \frac{v_{yi}}{g} + \sqrt{\frac{2h_{max}}{g}}$
 $= \frac{v_{yi}}{g} + \sqrt{\frac{2 \frac{v_{yi}^2}{2g}}{g}} = \frac{v_{yi}}{g} + \frac{v_{yi}}{g}$
 $= \frac{2v_{yi}}{g} \quad \text{!}$



$$d = v_x t + \frac{1}{2} a t^2$$

$$dy = v_{yi} t - \frac{1}{2} g t^2$$

$$0 = \frac{1}{2} g t^2 - v_{yi} t + dy$$

$$v_{fy} = v_{yi} + at$$

$$= v_{yi} - g \left(\frac{v_x}{g} + \frac{\sqrt{v_{yi}^2 - 2gdy}}{g} \right)$$

$$v_{fy} = \sqrt{v_{yi}^2 - 2gdy} \quad \therefore$$

$$v_i = v_{yi}$$

$$a = -g$$

$$d = dy$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{v_{yi} \pm \sqrt{v_{yi}^2 - 4(\frac{1}{2}g)dy}}{2(\frac{1}{2}g)}$$

$$= \frac{v_{yi} \pm \sqrt{v_{yi}^2 - 2gdy}}{g}$$

$$t = \frac{v_{yi}}{g} + \sqrt{\frac{2(h_{max} - dy)}{g}}$$

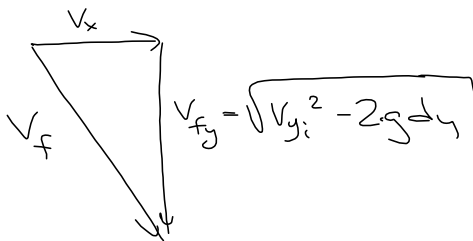
$$= \frac{v_{yi}}{g} + \sqrt{\frac{2h_{max} - 2dy}{g}}$$

$$= \frac{v_{yi}}{g} + \sqrt{\frac{v_{yi}^2}{g^2} - \frac{2dy}{g}}$$

$$t = \frac{v_{yi}}{g} \pm \sqrt{\frac{v_{yi}^2}{g^2} - \frac{2dy}{g}}$$

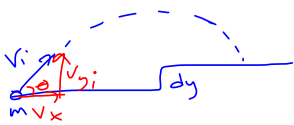
usually + root but need to check.

same \therefore



$$v_f^2 = v_x^2 + v_{yi}^2 - 2gdy$$

$$v_f^2 = v_i^2 - 2gdy$$



Solving Projectiles Using Energy

Kinetic Energy:

Beginning $\frac{1}{2}mv_i^2$

Top $\frac{1}{2}mv_x^2$

End $\frac{1}{2}mv_f^2$

Potential Energy:



Top mgh_{\max}

End $mgdy$

Total Energy:

Beginning $\frac{1}{2}mv_i^2$

Top $\frac{1}{2}mv_x^2$

End $\frac{1}{2}mv_i^2$

$$\frac{1}{2}mv_i^2 = mgdy + \frac{1}{2}mv_f^2$$

$$v_i^2 = 2gdy + v_f^2$$

$$v_f^2 = v_i^2 - 2gdy$$

