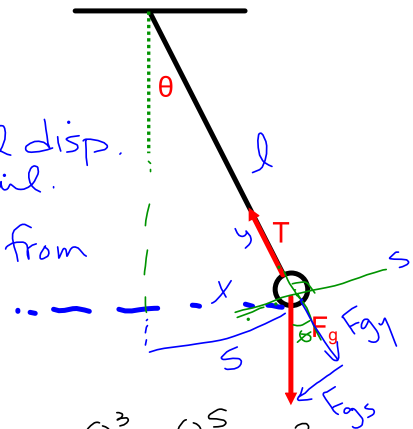
The Pendulum as a SHO

To be a SHO, recall F must be proportional to -x

Consider a pendulum of length, ℓ

x = horizontal disp. from equil.

s = arc length from equil.



Recall

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \ldots$$

θ small

$$\sin\theta \approx \theta$$

Assume θ small

$$x \approx s = \ell\theta$$

taking back to equil.

$$\Sigma F = -F_{g_s} = -mg\sin\theta$$

If θ small

$$\Sigma F \approx -mg\theta$$

$$\approx -mg\frac{x}{\ell}$$

$$F = -\frac{mg}{\ell}x$$

is of the form

$$F = -kx$$

with $k = \frac{mg}{\ell}$

Comparisons (Frequency and Period)

Pendulum

$$T = 2\pi\sqrt{\frac{\cancel{m}\,1}{\frac{\cancel{m}g}{l}}}$$

$$= 2\pi\sqrt{\frac{l}{g}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

$$v_0 = \sqrt{\frac{\cancel{m}g/l}{\cancel{m}}}\,A$$

$$v_0 = \sqrt{\frac{g}{l}}\,A$$

Spring

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$v_0 = \sqrt{\frac{k}{m}}\,A$$

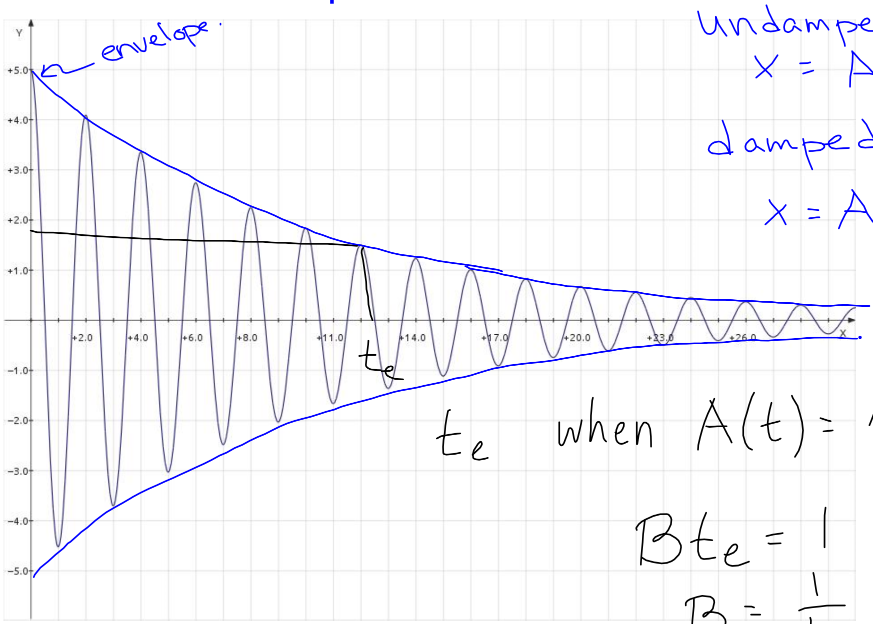
Circle

$$T = \frac{2\pi A}{v_0} \qquad v_0 = \sqrt{\frac{k}{m}}\,A$$

$$f = \frac{v_0}{2\pi A}.$$

$$v_0 = \frac{2\pi A}{T}$$

Real Life - Damped Harmonic Oscillators



undamped

$$x = A\cos 2\pi f t$$

damped

how quickly it damps depends on B

$$x = Ae^{-Bt}\cos 2\pi f t$$

t_e when $A(t) = \frac{A(0)}{e}$

$$Bt_e = 1$$

$$B = \frac{1}{t_e}$$

A note on e

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$\frac{de^x}{dx} = + 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \frac{4x^3}{4!} + \cdots$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = e^x$$

Quality factor of a spring

We define t_e to be the time for the amplitude of oscillation to drop to 1/e the initial amplitude.

$$A(t_e) = \frac{A(0)}{e} = \frac{A}{e} \qquad A(t) = A_0 e^{-\frac{t}{t_e}}$$

Then the quality factor, Q, is given by

$$Q = 2\pi f t_e.$$