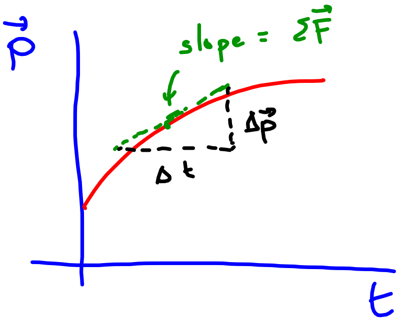
Momentum and Impulse

Newton's 2nd Law $\sum \vec{F} = m\vec{a}$ X

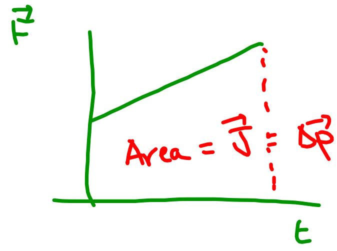
$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$



$$\sum \vec{F} \approx \frac{\Delta \vec{p}}{\Delta t}$$ ← non-calculus form.

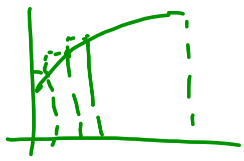
$$\vec{J} = \underbrace{\sum \vec{F} \Delta t}_{\text{Impulse}} = \Delta \vec{p}$$

Cause effect.



Calculus form

$$\vec{J} = \int_0^t \sum \vec{F}\, dt = \int_0^t d\vec{p}$$



But in the case where *m* is constant, we get

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$

$$= m\underbrace{\frac{d\vec{v}}{dt}}_{\vec{a}}$$

$$\sum \vec{F} = m\vec{a}$$

but $\vec{p} = m\vec{v}$

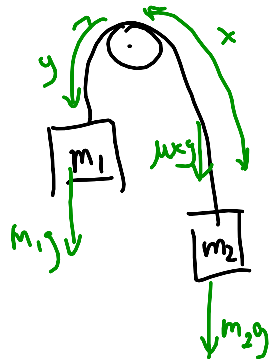
If m is constant

$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt}$$

But at any moment in time an object has a mass. So $\sum \vec{F} = m\vec{a}$ can be used at any <u>instant in time</u>.

$$m = \frac{|\sum \vec{F}|}{|\vec{a}|}$$ definition of inertial mass.

$x + y = l$



rope has a mass density of μ (mass per unit length) and length, l.

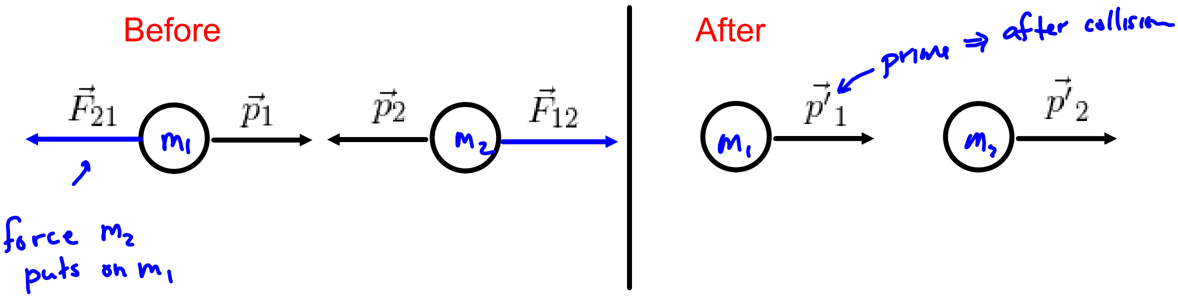
$$\sum F = (m_2 + \mu l_1 - m_1 - \mu l_2) g = (m_1 + m_2 + \mu l) \frac{d^2 x}{dt^2}$$

$$l_1 = l_1(t) = l_{10} + \frac{dx}{dt} t$$

$$l_2 = l_2(t) = l_{20} - \frac{dx}{dt} t$$

$$A \frac{d^2 x}{dt^2} + B \frac{dx}{dt} + C = ☺$$

Law of Conservation of Momentum Derivation



Consider two objects with momentum $\vec{p}_1$ and $\vec{p}_2$ respectively, which collide

$\vec{F}_{21} = -\vec{F}_{12}$ (Newton's 3rd Law)

$\vec{F}_{21}\Delta t = -\vec{F}_{12}\Delta t$ (Still 3rd law, Δts are the same)

$\Delta\vec{p}_1 = -\Delta\vec{p}_2$ (Newton's 2nd Law)

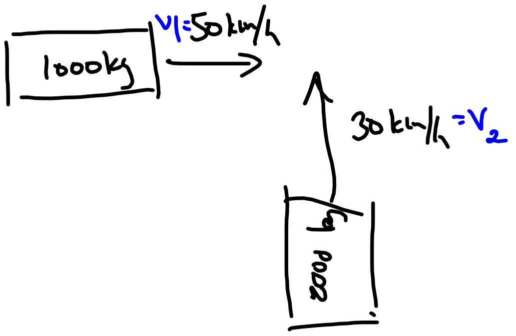
$$\vec{p}_1{}' - \vec{p}_1 = -(\vec{p}_2{}' - \vec{p}_2)$$
$$= -\vec{p}_2{}' + \vec{p}_2$$

$$\vec{p}_1{}' + \vec{p}_2{}' = \vec{p}_1 + \vec{p}_2$$

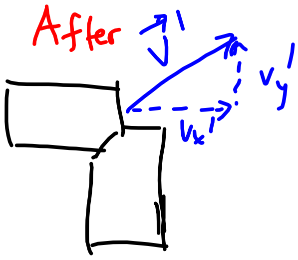
In general $\Sigma\vec{p} = \Sigma\vec{p}\,'$

Example 1

Before



After



$$\Sigma \vec{p} = \Sigma \vec{p}'$$

$$\Sigma \vec{p}_x = \Sigma \vec{p}_x'$$

$$m_1 v_1 = (m_1 + m_2) v_x'$$

$$(1000kg)(50km/h) = (1000 + 2000kg)\, v_x'$$

$$v_x' = 16.7\ km/h$$

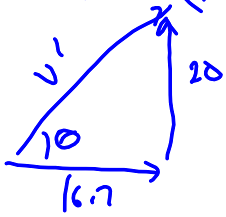
$$\Sigma \vec{p}_y = \Sigma \vec{p}_y'$$

$$m_2 v_2 = (m_1 + m_2) v_y'$$

$$2000(30) = (3000)\, v_y'$$

$$v_y' = 20\ km/h$$

$$\vec{v}' = (16.7\,\hat{x} + 20\,\hat{y})\ km/h$$



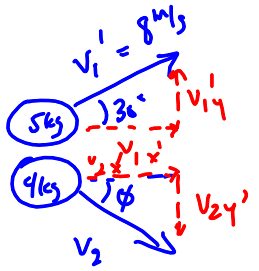
$$v' = \sqrt{v_x'^2 + v_y'^2} = \sqrt{(16.7)^2 + (20)^2}$$

$$= 26\ km/h$$

$$\tan\theta = \frac{20}{16.7}$$

$$\theta = 50^\circ$$

$\vec{v}' = 26\ km/h$ 50° ccw from +x axis, Example 2



$\vec{v}_2' = ?$

$$\Sigma \vec{p}_x = \Sigma \vec{p}_x'$$

$$m_1 v_1 = m_1 v_{1x}' + m_2 v_{2x}'$$

$$\Sigma \vec{p}_y = \Sigma \vec{p}_y' = \text{☺}$$

uppies = downies

$$m_1 v_{1y}' = m_2 v_{2y}'$$