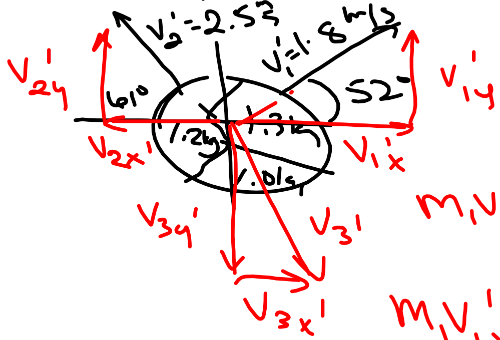
Homework questions?

p513 # 38

$\Sigma p = $ ☺

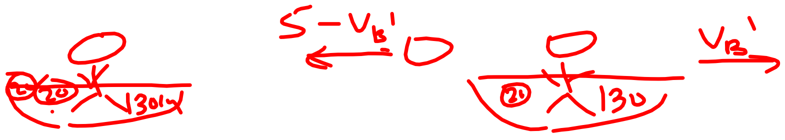
$\Sigma \vec{p}' = $ ☺



$m_1 v'_{1y} + m_2 v'_{2y} = m_3 v'_{3y}$

$m_1 v'_{1x} + m_3 v'_{3x} = m_2 v'_{2x}$

#3



1st rock.

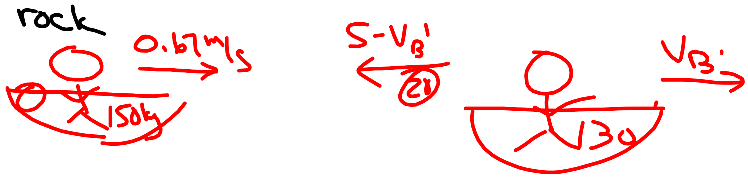
$$m v = m_R \vec{v}_R' + m_{B+R} \vec{v}_B'$$

$$0 = 150 v_B' - 20(5 - v_B')$$

$$100 = 170 v_B'$$

$$v_B' = 0.588 \frac{m}{s}$$

2nd rock



$$m v = m_B v_B' - m_R(5 - v_B')$$

$$150(0.588) = 130 v_B' - 20(5 - v_B')$$

$$= 150 v_B' - 100$$

$$v_B' = 1.25 \, m/s$$

Elastic vs. Inelastic Collisions

Goals:

- To recognize the difference between elastic and inelastic collisions
- To use conservation of kinetic energy to solve elastic collision problems
- To solve a system of 3 equations (with 3 unknowns) using conservation of momentum and conservation of kinetic energy for 2-D elastic collisions

Inelastic

- KE not conserved

Elastic

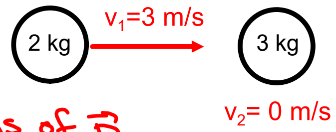
- KE conserved

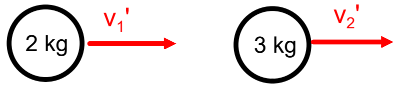
Completely inelastic

- maximum loss of KE → when objects stick together.

1-D Example of an Elastic Collision (Head on collision)

A 2 kg object travelling at 3 m/s strikes a 3 kg object at rest. If the collision is elastic and the 3 kg object travels in the same direction as the 2 kg object's initial velocity, what are the velocities of each object after the collision?





Cons of $\vec{p}$

$$\Sigma \vec{p} = \Sigma \vec{p}\,'$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \quad (v_2 = 0)$$

$$(2\,kg)(+3\,\tfrac{m}{s}) = (2\,kg)v_1' + (3\,kg)v_2'$$

① $$+6 = 2v_1' + 3v_2'$$

Cons of KE

$$\Sigma KE = \Sigma KE'$$

$$\tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 = \tfrac{1}{2} m_1 v_1'^2 + \tfrac{1}{2} m_2 v_2'^2 \quad (v_2 = 0)$$

$$2(3)^2 = 2v_1'^2 + 3v_2'^2$$

② $$18 = 2v_1'^2 + 3v_2'^2$$

Note: Quadratics have 2 possible solutions

One solution to these equations is that NOTHING happens

So $v_1' = +3\,\frac{m}{s}$ $v_2' = 0$ (Not the solution we are looking for, BUT we can use this knowledge to our advantage!)

It will be simple to solve the quadratic in v_2' since one solution is 0.

$$+6 = 2v_1' + 3v_2'$$

$$v_1' = 3 - \tfrac{3}{2}v_2'$$

$$18 = 2v_1'^2 + 3v_2'^2$$

$$18 = 2\left(3 - \tfrac{3}{2}v_2'\right)^2 + 3v_2'^2$$

$$18 = 2\left(9 - 9v_2' + \tfrac{9}{4}v_2'^2\right) + 3v_2'^2$$

$$18 = 18 - 18v_2' + \tfrac{9}{2}v_2'^2 + 3v_2'^2$$

$$0 = \tfrac{15}{2}v_2'^2 - 18v_2'$$

$$0 = 3v_2'\left(\tfrac{5}{2}v_2' - 6\right)$$

So $3v_2' = 0$ or $\frac{5}{2}v_2' - 6 = 0$

$v_2' = 0$ ← The "nothing happens" solution

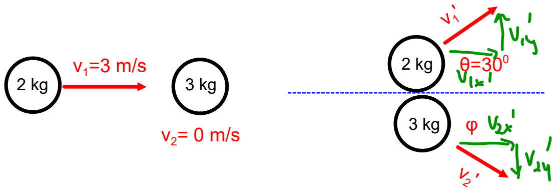
$v_2' = +2.4\,\frac{m}{s}$ ← The one we're looking for.

$$v_1' = 3 - \tfrac{3}{2}(+2.4)$$

$$= -0.6\,\tfrac{m}{s}$$

So m_1 rebounds at $0.6\,\frac{m}{s}$

A 2-D Example . Elastic collision
(Oblique Collision)



3 unknowns
- requires 3 equations
  - 2 momentum
  - 1 energy

$\Sigma \vec{p}_x = \Sigma \vec{p}_x{}'$

$m_1v_1 = m_1v_{1x}' + m_2v_{2x}'$

$2(3) = 2v_1'\cos 30^\circ + 3v_2'\cos\phi$

① $6 = \sqrt{3}\,v_1' + 3v_2'\cos\phi$

$0 = \Sigma \vec{p}_y = \Sigma \vec{p}_y{}'$

$m_1v_{1y}' = m_2v_{2y}'$

$2v_1'\sin 30^\circ = 3v_2'\sin\phi$ ($\sin 30^\circ = \frac{1}{2}$)

② $v_1' = 3v_2'\sin\phi$

$\Sigma KE = \Sigma KE'$

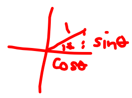
$\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$

$2(3)^2 = 2v_1'^2 + 3v_2'^2$

③ $18 = 2v_1'^2 + 3v_2'^2$

Advice: Eliminate ϕ first. (Avoid the obvious v_1' substitution - it's messy)

$\sin^2\phi + \cos^2\phi = 1$



$(6 - \sqrt{3}v_1')^2 = (3v_2'\cos\phi)^2$

$v_1'^2 = 9v_2'^2\sin^2\phi$

$36 - 12\sqrt{3}\,v_1' + 3v_1'^2 = 9v_2'^2\cos^2\phi$

$+ \quad v_1'^2 = 9v_2'^2\sin^2\phi$

$36 - 12\sqrt{3}\,v_1' + 4v_1'^2 = 9v_2'^2(\cos^2\phi + \sin^2\phi)$, where $(\cos^2\phi + \sin^2\phi) = 1$

$(18 = 2v_1'^2 + 3v_2'^2) \times 3$

$54 = 6v_1'^2 + 9v_2'^2$

$54 - 6v_1'^2 = 9v_2'^2$

$36 - 12\sqrt{3}\,v_1' + 4v_1'^2 = 54 - 6v_1'^2$

$10v_1'^2 - 12\sqrt{3} - 18 = 0$

$5v_1'^2 - 6\sqrt{3} - 9 = 0$

$v_1' = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{6\sqrt{3} \pm \sqrt{108 - 4(5)(-9)}}{10}$

$= \dfrac{6\sqrt{3} \pm 12\sqrt{2}}{10}$

Note: The - sol'n gives a - value for v_1'. This is not a valid physical solution.

So $v_1' = \dfrac{3\sqrt{3} + 6\sqrt{2}}{5}$

$v_1' = 2.74\,\frac{m}{s}$

$18 = 2v_1'^2 + 3v_2'^2$

$18 = 2(2.74)^2 + 3v_2'^2$

$v_2' = 0.994\,m/s$

$v_1' = 3v_2'\sin\phi$

$2.74\,\frac{m}{s} = 3(0.994\,m/s)\sin\phi$

$\sin\phi = \dfrac{2.74}{2.982} = 0.918$

$\phi = 67^\circ$

Homework:
Sheet # 4, 5
Text p. 515 # 39, 40