Homework questions?

Elastic vs. Inelastic Collisions

Goals:

- To recognize the difference between elastic and inelastic collisions
- To use conservation of kinetic energy to solve elastic collision problems
- To solve a system of 3 equations (with 3 unknowns) using conservation of momentum and conservation of kinetic energy for 2-D elastic collisions

Inelastic

- momentum conserved but KE is NOT.

Elastic

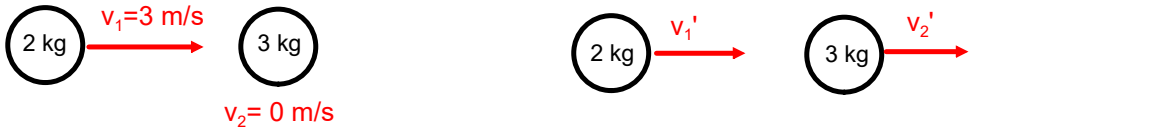
- momentum conserved
- KE conserved.

Completely Inelastic

- max KE loss when the objects stick ~~together~~

1-D Example of an Elastic Collision (Head on collision)

A 2 kg object travelling at 3 m/s strikes a 3 kg object at rest. If the collision is elastic and the 3 kg object travels in the same direction as the 2 kg object's initial velocity, what are the velocities of each object after the collision?



$\Sigma \vec{p} = \Sigma \vec{p}'$

$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$

$2(+3) = 2v_1' + 3v_2'$

① $\boxed{+6 = 2v_1' + 3v_2'}$

$\Sigma KE = \Sigma KE'$

$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$

$2(3)^2 = 2v_1'^2 + 3v_2'^2$

$\boxed{18 = 2v_1'^2 + 3v_2'^2}$ ②

Note: To solve these we are solving a quadratic which has 2 solutions (for each variable). One of the solutions is that nothing happens, i.e., $v_1' = 3\frac{m}{s}$, $v_2' = 0$! (This is not the solution we are looking for BUT we can use this knowledge to our advantage!)

By solving our quadratic in v_2' (which has one solution of zero) we make the math easier.

$6 - 3v_2' = 2v_1'$

$v_1' = 3 - \frac{3}{2}v_2'$

$18 = 2v_1'^2 + 3v_2'^2$

$18 = 2\left(3 - \frac{3}{2}v_2'\right)^2 + 3v_2'^2$

$18 = 2\left(9 - 9v_2' + \frac{9}{4}v_2'^2\right) + 3v_2'^2$

$18 = 18 - 18v_2' + \frac{9}{2}v_2'^2 + 3v_2'^2$

$0 = \frac{15}{2}v_2'^2 - 18v_2'$

$0 = v_2'\left(\frac{15}{2}v_2' - 18\right)$

"nothing happens" $v_2' = 0$ sol'n

the one we want

$0 = \frac{15}{2}v_2' - 18$

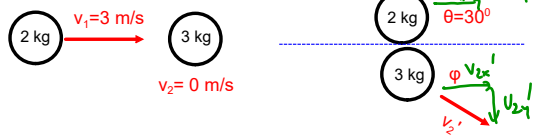
$v_2' = 2.4\ m/s$

$v_1' = 3 - \frac{3}{2}(2.4\ m/s)$

$= -0.6\ \frac{m}{s}$

So the 2kg mass rebounds at $0.6\ \frac{m}{s}$

A 2-D Example (Oblique collision)
Assume elastic



3 unknowns (v_1', v_2', φ)
3 equations
- 1 from cons of KE
- 2 from $\vec{p}$ (x, y)

$\Sigma \vec{p}_x = \Sigma \vec{p}_x'$ $\quad$ $\Sigma \vec{p}_y = \Sigma \vec{p}_y'$ $\quad$ $\Sigma KE = \Sigma KE'$

$m_1 v_1 = m_1 v_{1x}' + m_2 v_{2x}'$ $\quad$ $m_1 v_{1y}' = m_2 v_{2y}'$ $\quad$ $\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$

$2(3) = 2v_1' \cos 30° + 3v_2' \cos\varphi$ $\quad$ $2v_1' \sin 30° = 3v_2' \sin\varphi$ $\quad$ $2(3)^2 = 2v_1'^2 + 3v_2'^2$

① $6 = \sqrt{3} v_1' + 3v_2' \cos\varphi$ $\quad$ ② $v_1' = 3v_2' \sin\varphi$ $\quad$ ③ $18 = 2v_1'^2 + 3v_2'^2$

Advice: Resist the urge to make the "obvious" substitution. Eliminate φ first.

$\sin^2\varphi + \cos^2\varphi = 1$

$v_1'^2 = 9v_2'^2 \sin^2\varphi$

$(6 - \sqrt{3} v_1')^2 = (3v_2' \cos\varphi)^2$

$36 - 12\sqrt{3} v_1' + 3v_1'^2 = 9v_2'^2 \cos^2\varphi$

$+ \quad v_1'^2 = 9v_2'^2 \sin^2\varphi$

$36 - 12\sqrt{3} v_1' + 4v_1'^2 = 9v_2'^2 (\sin^2\varphi + \cos^2\varphi)$

$(18 = 2v_1'^2 + 3v_2'^2) \times 3$

$54 - 6v_1'^2 = 9v_2'^2$

$36 - 12\sqrt{3} v_1' + 4v_1'^2 = 54 - 6v_1'^2$

$10v_1'^2 - 12\sqrt{3} v_1' - 18 = 0$ ☺

$5v_1'^2 - 6\sqrt{3} v_1' - 9 = 0$ ☺

$v_1' = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{6\sqrt{3} \pm \sqrt{108 - 4(5)(-9)}}{10}$

$= \dfrac{6\sqrt{3} \pm \sqrt{288}}{10}$ $\quad$ $\sqrt{288} > 6\sqrt{3}$ $\quad$ $12\sqrt{2} > 6\sqrt{3}$

So − solution produces a $-v_1'$ not a valid physical sol'n.

$= \dfrac{6\sqrt{3} + 12\sqrt{2}}{10}$

$= 2.74 \text{ m/s}$ $\Longrightarrow$ $18 = 2v_1'^2 + 3v_2'^2$

$18 = 2(2.74)^2 + 3v_2'^2$

$v_2' = 0.997 \frac{m}{s}$

$v_1' = 3v_2' \sin\varphi$

$2.74 = 3(0.997) \sin\varphi$

$\varphi = 66°$

$\Sigma KE = \frac{1}{2} m_1 v_1^2$

$= 9$

$\Sigma KE' = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$

$= \frac{1}{2}(2(2.74)^2 + 3(0.997)^2)$

$= 8.9986$

$9 < 8.99 \approx 8.82$

Homework:
Sheet # 4, 5
Text p. 515 # 39, 40