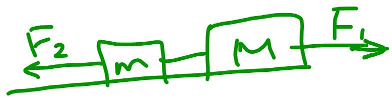
Day 4 - The Power of Free Bodies   Section 4-7



Consider a simple Atwood machine with M > m

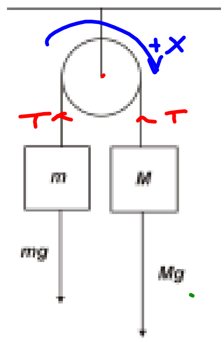


Figure 2.4: Atwood machine

$$\Sigma F = Mg - mg = \text{"}m\text{"}\, a$$ ← total mass of system

$$(M-m)g = (M+m)\,a$$

$$\vec{a} = \frac{M-m}{M+m}\, g\, \hat{x}$$

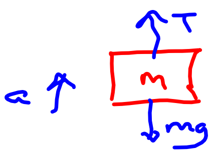
We have assumed $M_{axle} = 0$

string doesn't stretch

$(M_{pulley} = m_{string} = 0)$

M_{pulley}, m_{string} $\ll M, m$

FBD



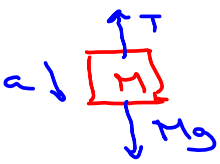
$$\Sigma F = T - mg = ma$$ ← only m!

$$T = mg + m\left(\frac{M-m}{M+m}\right)g$$

$$= mg\left(1 + \frac{M-m}{M+m}\right)$$

$$= mg\left(\frac{M+m+M-m}{M+m}\right)$$

$$T = \frac{2Mm}{M+m}\, g$$



$$\Sigma F = Mg - T = Ma$$

$$T = Mg - M\left(\frac{M-m}{M+m}\right)g$$

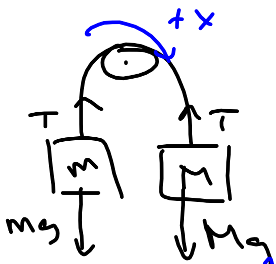
$$= Mg\left(\frac{M+m-(M-m)}{M+m}\right)$$

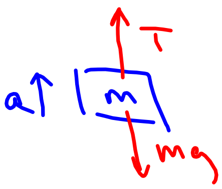
$$= \frac{2Mm}{M+m}\, g$$

Same ☺

Alternate solution

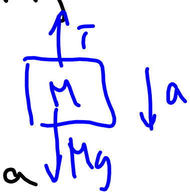
Two FBDs





$$\sum F = T - mg = ma$$

$$T = ma + mg$$



$$\sum F = Mg - T = Ma$$

$$\sum F = Mg - (ma + mg) = Ma$$

$$= (M - m)g = (M + m)a$$

$$a = \frac{M - m}{M + m} g$$

Can subst. a into either equation to get T.

But let's change the question slightly. In this case, let's say we have mass M connected to the end of a rope that will only hold a weight, T (we use T for tension), before it breaks. What is the maximum mass, m, that we can attach to the system without the rope breaking? ← same $\vec{a}$

What is the acceleration?

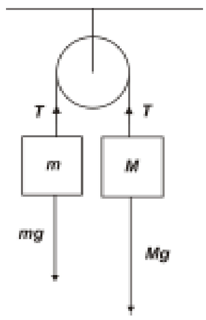


Figure 2.5: Atwood machine

So we can use what we have done

$$T = \frac{2Mm}{M+m} g$$

$$MT + mT = 2Mmg$$

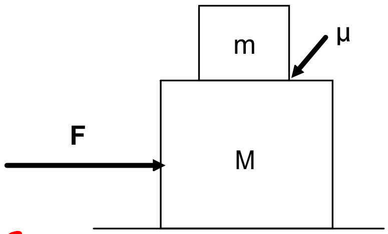
$$mT - 2Mmg = -MT$$

$$m(T - 2Mg) = -MT$$

$$m = \frac{MT}{2Mg - T} = \frac{T}{2Mg - T} M$$

Now, consider the situation at right. . .

What is the maximum force that can be applied to M without m slipping (you may assume the coefficient of friction between M and the ground is negligible)?

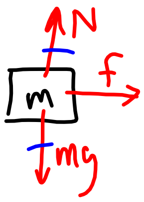


→ same $\vec{a}$

$$\Sigma F = F = (M+m)a$$

$$a = \frac{F}{M+m}$$

FBD of little m (fewer force ∴ easier)



$$\Sigma F = f = ma$$

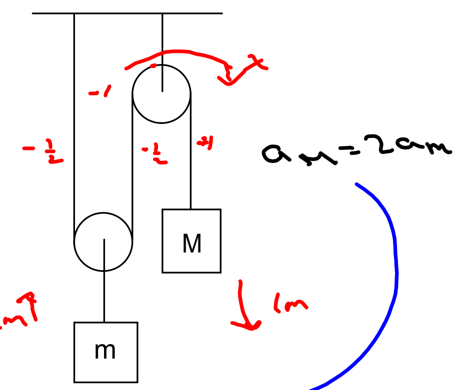
$$f_{min} = \frac{m}{M+m} F$$

$$\mu_{min} N = \frac{m}{M+m} F$$

$$\mu_{min} mg = \frac{m}{M+m} F$$

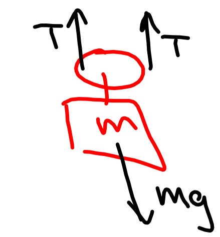
$$\mu_{min} = \frac{F}{(M+m)g}$$

What is the acceleration of m (assuming M is sufficiently large to accelerate m upward and the pulleys are massless)?



$a_M = 2a_m$

Need FBDs



$\Sigma F = 2T - mg = m a_m$

$\Sigma F = Mg - T = M a_M$

$Mg - 2M a_m = T$

$2(Mg - 2M a_m) - mg = m a_m$