Example

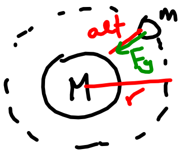
A satellite of mass 1000 kg is placed in orbit above Mars at an altitude of 3.0 x 10^6 m. Using data from one of Mars' moons and the radius of Mars, determine:

a) The orbital period of the satellite,
b) The orbital speed of the satellite,
c) The acceleration of the satellite,
d) The weight of the satellite in orbit, and
e) The mass of Mars.

$r_p = 9.30 \times 10^6$ m   25200 / 2340

$T_p = 7\text{h } 39\text{ min} = (7 \times 3600 + 39 \times 60)$

$= 2.754 \times 10^4$ s



alt = 3.0×10^6 m

$r_{sat} = r_M + alt$

$= 3.40 \times 10^6 + 3.0 \times 10^6$ m

$= 6.4 \times 10^6$ m

a) $\frac{r^3_{Phobos}}{T^2_{Phobos}} = \frac{r^3_{sat}}{T^2_{sat}}$

$\frac{(9.30 \times 10^6)^3}{(2.75 \times 10^4 s)^2} = \frac{(6.4 \times 10^6)^3}{T_{sat}^2}$

$T_{sat} = \sqrt{\frac{6.4^3}{9.3^3}} \cdot 2.75 \times 10^4$ s

a) $= 1.57 \times 10^4$ s

b) $v = \frac{2\pi r}{T}$

$= \frac{2\pi (6.4 \times 10^6) m}{1.57 \times 10^4 s}$

$= 2.56 \times 10^3 \frac{m}{s}$

c) $a_c = \frac{v^2}{r}$

$= \frac{(2.56 \times 10^3)^2}{6.4 \times 10^6}$

$= 1.0$ m/s^2

d) $F_g = F_c = m a_c$

$= 1000$ N

e) $F_g = \frac{GMm}{r^2} = \frac{mv^2}{r}$

so $\frac{GM}{r^2} = a_c = 1.0$

$M = \frac{1.0 \cdot (6.4 \times 10^6)^2}{6.67 \times 10^{-11}}$

$= 6.14 \times 10^{23}$ kg

Note: Since F_g is the ONLY force:

$F_g = \Sigma F = F_c$

$a_g = a = a_c$

Note 2: acceleration due to gravity is also called the "gravitational field strength."

Universal Gravitation

1. A satellite orbits around Earth at a distance of 1.28×10^7 m from the center of the Earth. The satellite weighs 6000 N on the surface of Earth. For the satellite in orbit calculate its
   (a) mass. (612 kg)
   (b) weight. (1.49×10^3 N)
   (c) speed. ($5.58 \times 10^3\ \frac{m}{s}$)
2. A satellite which weighs 1.0×10^4 N on the surface of Earth is put into circular orbit 7.05×10^8 m above the Earths surface. Calculate its
   (a) mass (1.0×10^3 kg)
   (b) weight (0.79 N)
   (c) velocity ($7.5 \times 10^2\ \frac{m}{s}$)
   (d) acceleration towards the Earth. ($7.9 \times 10^{-4}\ \frac{m}{s^2}$)
3. A satellite orbits Neptune in 200 minutes. The radius of its orbit is 2.92×10^7 s. Calculate
   (a) the average speed of the satellite. ($1.53 \times 10^4\ \frac{m}{s}$)
   (b) its centripetal acceleration. ($8.01\ \frac{m}{s^2}$)
4. What orbital speed must a satellite of mass 800 kg have in order to maintain an orbit 2.00×10^7 m above the surface of Jupiter where the gravitational field strength is $15\ \frac{m}{s^2}$? What would it weigh at this height? ($3.7 \times 10^4\ \frac{m}{s}$, 1.2×10^4 N)
5. Compute the gravitational force between a proton and an electron using the following data:
   mass of proton = 1.67×10^{-27} kg
   mass of electron = 9.11×10^{-31} kg
   radius of orbit of an electron = 5.29×10^{-9} cm. (3.63×10^{-47} N)

6. A space explorer is 1 billion km away from a certain star and she observes that the gravitational force between herself and the star is 1000 N. What will this force be when she is half a billion km from the star?
(4000 N)

7. A satellite circles the Earth once every 95 minutes at an average altitude of 500 km. Calculate the mass of the Earth. (5.9×10^{24} kg)

8. A satellite put into circular orbit around Uranus weighs 2.0×10^4 N on Earth. The radius of the satellites orbit is 4.0×10^7 m (DO NOT use the mass of Uranus in your calculations). Calculate
   (a) the period of the satellite. (2.1×10^4s)
   (b) its orbital velocity. (1.2×10^4 $\frac{m}{s}$)
   (c) the force needed to maintain this orbit. (7.2×10^3N)
   (d) the centripetal acceleration (3.6 $\frac{m}{s^2}$)
   (e) the mass of Uranus. (8.6×10^{25}kg)

9. A satellite which weighs 7.0×10^3 N on Earth is put into orbit 200 km above the surface of Mars. For the satellite find the
   (a) mass. (7.1×10^2 kg)
   (b) weight in orbit. (2.3×10^3 N)
   (c) gravitational field strength acting on it. (3.2 $\frac{m}{s^2}$)
   (d) speed of the satellite. (3.4×10^3 $\frac{m}{s}$)

10. A satellite with a mass of 640 kg is in orbit above the surface of the Earth where the gravitational field strength is 8.6 $\frac{m}{s^2}$. What is the gravitational force on the satellite at this height? (5.5×10^3 N)

11. A 1000 kg satellite is put into a circular orbit above Earth so that it always remains over the same place on Earth. (This is called a synchronous or geostationary orbit.)
    (a) What is the radius of this orbit? (4.22×10^7m)
    (b) What would the satellite weigh in orbit? (224 N)
    (c) How fast does it go while orbiting? (3.07×10^3 $\frac{m}{s}$)