

Example

A satellite of mass 1000 kg is placed in orbit above Mars at an altitude of 3.0×10^6 m. Using data from one of Mars' moons and the radius of Mars, determine:

- a) The orbital period of the satellite,
- b) The orbital speed of the satellite,
- c) The acceleration of the satellite,
- d) The weight of the satellite in orbit, and
- e) The mass of Mars.

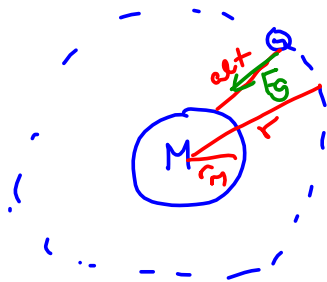
25200
 25×10^3

$r_{\text{Phobos}} = 9.30 \times 10^6 \text{ m}$

$T_P = 7 \text{ h } 39 \text{ min}$
 $= (7 \times 3600 + 39 \times 60 \text{ s})$
 $= \underline{\underline{27540 \text{ s}}}$

$m = 1000 \text{ kg}$
 $\text{alt} = 3.0 \times 10^6 \text{ m}$

$r_M = 3.40 \times 10^6 \text{ m}$
 $r_{\text{sat}} = r_M + \text{alt} = \underline{\underline{6.4 \times 10^6 \text{ m}}}$



c) $\frac{r_P^3}{T_P^2} = \frac{r_{\text{sat}}^3}{T_{\text{sat}}^2}$

$\frac{(9.30 \times 10^6)^3}{(27540)^2} = \frac{(6.4 \times 10^6)^3}{T_{\text{sat}}^2}$

a) $T_{\text{sat}} = \sqrt{\left(\frac{6.4}{9.3}\right)^3} 27540 \text{ s}$
 $= \underline{\underline{15722 \text{ s}}}$

b) $v = \frac{2\pi r}{T}$
 $= \frac{2\pi (6.4 \times 10^6 \text{ m})}{1.57 \times 10^4 \text{ s}}$
 $= 25.6 \times 10^2$
 $= \underline{\underline{2.56 \times 10^3 \text{ m/s}}}$

c) $a_c = \frac{v^2}{r}$
 $= \frac{(2.56 \times 10^3)^2}{6.4 \times 10^6}$
 $= \underline{\underline{1.0 \text{ m/s}^2}}$

Note: $F_g = \Sigma F = F_c$
Since F_g is the ONLY force. So
 $a_g = a = a_c$

Note 2: a_g is also known as "gravitational field strength."

d) $F_g = \Sigma F = F_c = \frac{mv^2}{r} = ma_c$
 $= (1000 \text{ kg})(1.0 \text{ m/s}^2)$
 $= \underline{\underline{1000 \text{ N}}}$

e) $F_g = \frac{GMm}{r^2} = ma_g$

$M = \frac{a_g r^2}{G}$
 $= \frac{(1.0 \text{ m/s}^2)(6.4 \times 10^6)^2}{6.67 \times 10^{-11}}$
 $= \underline{\underline{6.2 \times 10^{23} \text{ kg}}}$

Universal Gravitation

1. A satellite orbits around Earth at a distance of 1.28×10^7 m from the center of the Earth. The satellite weighs 6000 N on the surface of Earth. For the satellite in orbit calculate its
 - (a) mass. (612 kg)
 - (b) weight. (1.49×10^3 N)
 - (c) speed. ($5.58 \times 10^3 \frac{m}{s}$)
2. A satellite which weighs 1.0×10^4 N on the surface of Earth is put into circular orbit 7.05×10^8 m above the Earth's surface. Calculate its
 - (a) mass (1.0×10^3 kg)
 - (b) weight (0.79 N)
 - (c) velocity ($7.5 \times 10^2 \frac{m}{s}$)
 - (d) acceleration towards the Earth. ($7.9 \times 10^{-4} \frac{m}{s^2}$)
3. A satellite orbits Neptune in 200 minutes. The radius of its orbit is 2.92×10^7 s. Calculate
 - (a) the average speed of the satellite. ($1.53 \times 10^4 \frac{m}{s}$)
 - (b) its centripetal acceleration. ($8.01 \frac{m}{s^2}$)
4. What orbital speed must a satellite of mass 800 kg have in order to maintain an orbit 2.00×10^7 m above the surface of Jupiter where the gravitational field strength is $15 \frac{m}{s^2}$? What would it weigh at this height? ($3.7 \times 10^4 \frac{m}{s}$, 1.2×10^4 N)
5. Compute the gravitational force between a proton and an electron using the following data:

mass of proton = 1.67×10^{-27} kg
 mass of electron = 9.11×10^{-31} kg
 radius of orbit of an electron = 5.29×10^{-9} cm. (3.63×10^{-47} N)

6. A space explorer is 1 billion km away from a certain star and she observes that the gravitational force between herself and the star is 1000 N. What will this force be when she is half a billion km from the star?
(4000 N)
7. A satellite circles the Earth once every 95 minutes at an average altitude of 500 km. Calculate the mass of the Earth.
(5.9×10^{24} kg)
8. A satellite put into circular orbit around Uranus weighs 2.0×10^4 N on Earth. The radius of the satellite's orbit is 4.0×10^7 m (DO NOT use the mass of Uranus in your calculations). Calculate
- the period of the satellite. (2.1 $\times 10^4$ s)
 - its orbital velocity. ($1.2 \times 10^4 \frac{\text{m}}{\text{s}}$)
 - the force needed to maintain this orbit. (7.2×10^3 N)
 - the centripetal acceleration ($3.6 \frac{\text{m}}{\text{s}^2}$)
 - the mass of Uranus. (8.6×10^{26} kg)
9. A satellite which weighs 7.0×10^3 N on Earth is put into orbit 200 km above the surface of Mars. For the satellite find the
- mass. (7.1×10^2 kg)
 - weight in orbit. (2.3×10^3 N)
 - gravitational field strength acting on it. ($3.2 \frac{\text{m}}{\text{s}^2}$)
 - speed of the satellite. ($3.4 \times 10^3 \frac{\text{m}}{\text{s}}$)
10. A satellite with a mass of 640 kg is in orbit above the surface of the Earth where the gravitational field strength is $8.6 \frac{\text{m}}{\text{s}^2}$. What is the gravitational force on the satellite at this height?
(5.5×10^3 N)
11. A 1000 kg satellite is put into a circular orbit above Earth so that it always remains over the same place on Earth. (This is called a synchronous or geostationary orbit.)
- What is the radius of this orbit? (4.22×10^7 m)
 - What would the satellite weigh in orbit? (224 N)
 - How fast does it go while orbiting? ($3.07 \times 10^3 \frac{\text{m}}{\text{s}}$)