Elastic Collisions

- Is an interaction in which kinetic energy is conserved.
- Note 1: This rarely happens on the macroscopic scale, unless there is no contact between the objects.
- Note 2: (Almost) All submicroscopic "collisions" are elastic (e.g. the interactions between atoms, subatomic particles, etc.)
- Note 3: If 2 objects stick together, KE is NOT conserved, and in fact, represents the greatest KE loss. For this reason, these collisions are sometimes called "completely inelastic".

Note 4: Do not assume any macroscopic collision IS elastic!!

1-D (Head-on collision) Elastic Collision

$\Sigma \vec{p} = \Sigma \vec{p}\,'$

$m_1\vec{v}_1 = m_1\vec{v}_1{}' + m_2\vec{v}_2{}'$

$2(+3) = 2v_1' + 3v_2'$

① $+6 = 2v_1' + 3v_2'$

$\Sigma KE = \Sigma KE'$

$\cancel{\tfrac{1}{2}} m_1 v_1^2 = \cancel{\tfrac{1}{2}} m_1 v_1'^2 + \cancel{\tfrac{1}{2}} m_2 v_2'^2$

$2(3)^2 = 2v_1'^2 + 3v_2'^2$

$18 = 2v_1'^2 + 3v_2'^2$ ②

One of the solutions of these 2 equations must be

$v_1' = +3\frac{m}{s}$, $v_2' =$ ☺ (i.e. nothing happens)

So if we have to solve a quadratic equation I would rather solve one whose answer is ☺, i.e, v_2'

Solving ① for v_1'

$2v_1' = 6 - 3v_2'$

$v_1' = 3 - \frac{3}{2}v_2'$

Substituting into ②

$18 = 2\left(3 - \frac{3}{2}v_2'\right)^2 + 3v_2'^2$

$18 = 2\left(9 - 9v_2' + \frac{9}{4}v_2'^2\right) + 3v_2'^2$

☺ $\cancel{18} = \cancel{18} - 18v_2' + \frac{15}{2}v_2'^2$

☺ $= \frac{15}{2}v_2'^2 - 18v_2'$

$= 3v_2'\left(\frac{5}{2}v_2' - 6\right)$

So $v_2' =$ ☺ OR $\frac{5}{2}v_2' - 6 =$ ☺

$v_2' = +2.4\frac{m}{s}$

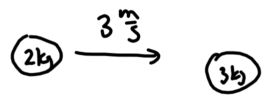
back into ①

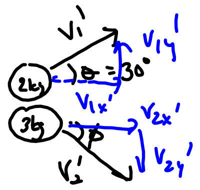
$v_1' = 3 - \frac{3}{2}(+2.4)$

$= -0.6\frac{m}{s}$

(so m_1 bounces backwards)

2D (oblique) elastic collision





$\Sigma p_x = \Sigma p_x'$

$m_1 v_1 = m_1 v_{1x}' + m_2 v_{2x}'$

$6 = 2\, v_1' \cos 30° + 3 v_2' \cos\phi$

① $6 = \sqrt{3}\, v_1' + 3 v_2' \cos\phi$

$\Sigma p_y = \Sigma p_y' = 0$

$0 = m_1 v_{1y}' - m_2 v_{2y}'$

$2 v_1' \sin 30° = 3 v_2' \sin\phi$

② $v_1' = 3 v_2' \sin\phi$

$\Sigma KE = KE'$

$\cancel{\tfrac{1}{2}} m_1 v_1^2 = \cancel{\tfrac{1}{2}} m_1 v_1'^2 + \cancel{\tfrac{1}{2}} m v_2'^2$

③ $18 = 2 v_1'^2 + 3 v_2'^2$

Note: resist every urge to eliminate v_1' (or v_2') first! Eliminate angles FIRST.

Rearrange ① and square

$(6 - \sqrt{3}\, v_1')^2 = (3 v_2' \cos\phi)^2$

$(6 - \sqrt{3} v_1')^2 = 9 v_2'^2 \cos^2\phi$

Square ②

$v_1'^2 = 9 v_2'^2 \sin^2\phi$

ADD

$(6 - \sqrt{3} v_1')^2 + v_1'^2 = 9 v_2'^2 (\sin^2\phi + \cos^2\phi)$

④ $9 v_2'^2 = 36 - 12\sqrt{3}\, v_1' + 4 v_1'^2$

③ $18 = 2 v_1'^2 + 3 v_2'^2$

Multiply ③ ×3

$54 = 6 v_1'^2 + 9 v_2'^2$

④ into ③

$54 = 6 v_1'^2 + 36 - 12\sqrt{3}\, v_1' + 4 v_1'^2$

$0 = 10 v_1'^2 - 12\sqrt{3}\, v_1' - 18$

$0 = 5 v_1'^2 - 6\sqrt{3}\, v_1' - 9$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

neg sol'n gives $-v_1'$ only + sol'n is valid

$v_1' = \dfrac{6\sqrt{3} \pm \sqrt{108 - 4(5)(-9)}}{10}$

$= \dfrac{6\sqrt{3} + 12\sqrt{2}}{10}$ 4.56

$= 2.74$ m/s

Can subst this back into eq ③ for v_2'

and into ② for ϕ