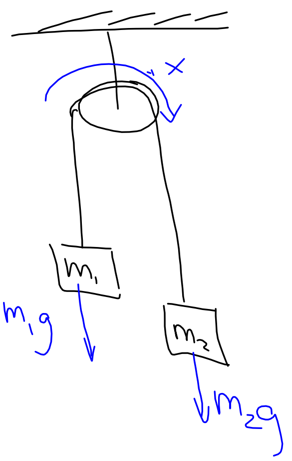
Pulleys

RECALL: ATWOOD MACHINE

ASSUME PULLEY, ROPE ARE LIGHT.

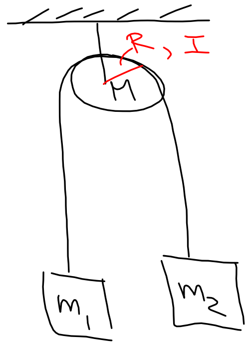


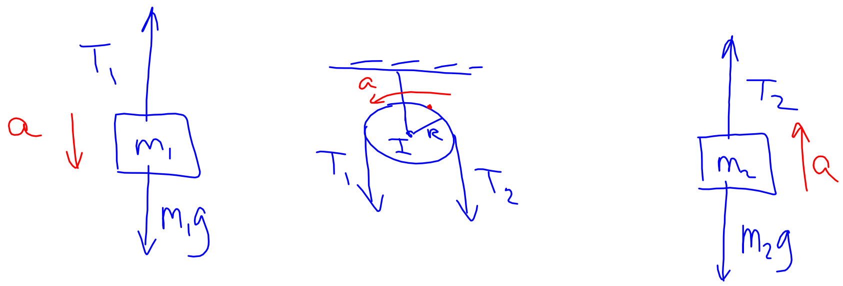
$$\Sigma F = (m_2 - m_1) g = (m_1 + m_2) a$$

$$\vec{a} = \frac{m_2 - m_1}{m_1 + m_2} g \ \hat{x}$$

BUT WHAT IF PULLEY ISN'T LIGHT?

ASSUME ROPE IS LIGHT, NO FRICTION ON THE AXLE OF THE PULLEY, NO SLIPPAGE BETWEEN ROPE AND PULLEY





$$m_1 g - T_1 = m_1 a \qquad \Sigma\tau = T_1 R - T_2 R = I\alpha \qquad T_2 - m_2 g = m_2 a$$

$$T_1 = m_1 g - m_1 a \qquad (T_1 - T_2)R = I\frac{a}{R} \qquad T_2 = m_2 g + m_2 a$$

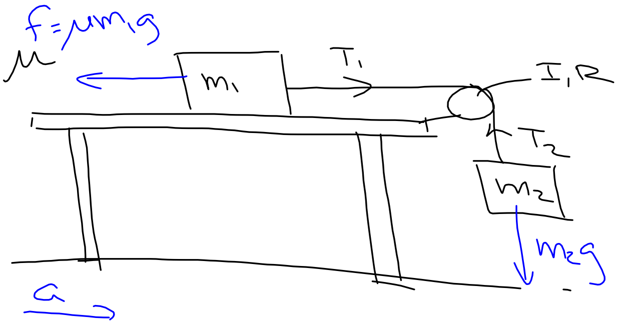
$$(m_1 g - m_1 a - m_2 g - m_2 a)R^2 = I a$$

$$(m_1 g - m_2 g)R^2 = (I + m_1 R^2 + m_2 R^2)\, a$$

$$a = \frac{m_1 - m_2}{I + m_1 R^2 + m_2 R^2} R^2 g$$

$$= \frac{m_1 - m_2}{I/R^2 + m_1 + m_2} g$$

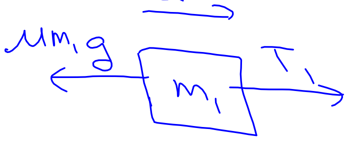
Find a, T_1, T_2.



$\Sigma F = ma$

$(m_2 - \mu m_1) g = (m_1 + m_2 + \frac{I}{R^2}) a$

$$a = \frac{m_2 - \mu m_1}{m_1 + m_2 + \frac{I}{R^2}} g$$

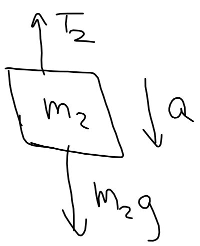


$$T_1 - \mu m_1 g = m_1 \left(\frac{m_2 - \mu m_1}{m_1 + m_2 + \frac{I}{R^2}}\right) g$$

$$T_1 = m_1 g \left(\frac{m_2 - \mu m_1}{m_1 + m_2 + \frac{I}{R^2}} + \mu\right)$$

$$= m_1 g \left(\frac{m_2 - \mu m_1 + \mu m_1 + \mu m_2 + \mu \frac{I}{R^2}}{m_1 + m_2 + \frac{I}{R^2}}\right)$$

$$= m_1 g \left(\frac{m_2 + \mu m_2 + \mu \frac{I}{R^2}}{m_1 + m_2 + \frac{I}{R^2}}\right)$$



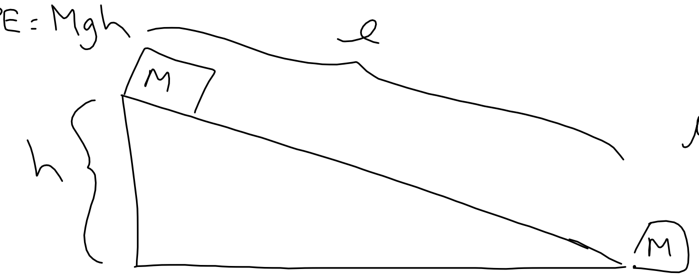
$m_2 g - T_2 = m_2 (\text{mess})$

$$T_2 = m_2 g \left(\frac{m_2 - \mu m_1}{m_1 + m_2 + I/R^2}\right)$$

Rotational Energy

RECALL:

$PE = Mgh$



$\mu = $ ☺

$v = ?$ $KE = \frac{1}{2} M V_f^2$

$$\frac{1}{2} \cancel{M} v_f^2 = \cancel{M} g h$$

$$V_f = \sqrt{2gh}$$

$I = MR^2$

Assume
No slippage

$KE = \frac{1}{2} M v_f^2$

$RE = \frac{1}{2} I \omega_f^2$

Rotational Kinetic Energy

$RE = \frac{1}{2} I \omega^2$

$$Mgh = \frac{1}{2} M v_f^2 + \frac{1}{2} I \omega_f^2 \qquad \omega_f = \frac{v}{R}$$

$$= \frac{1}{2} M v_f^2 + \frac{1}{2}(MR^2)\frac{v_f^2}{R^2}$$

$$\cancel{M} gh = \cancel{M} v_f^2$$

$$v_f = \sqrt{gh}$$