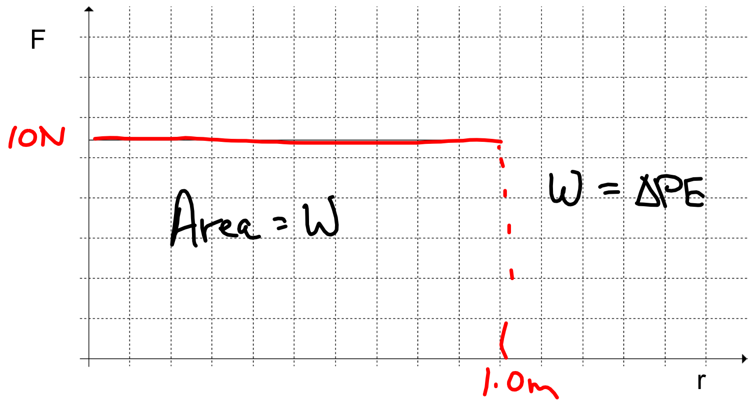
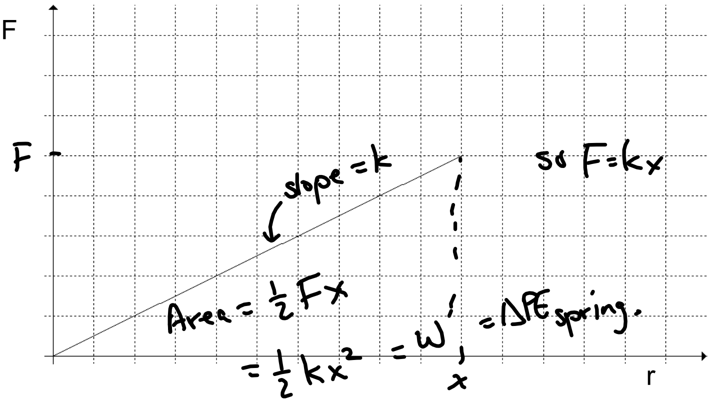
Gravitational Potential Energy

Recall W=Fd



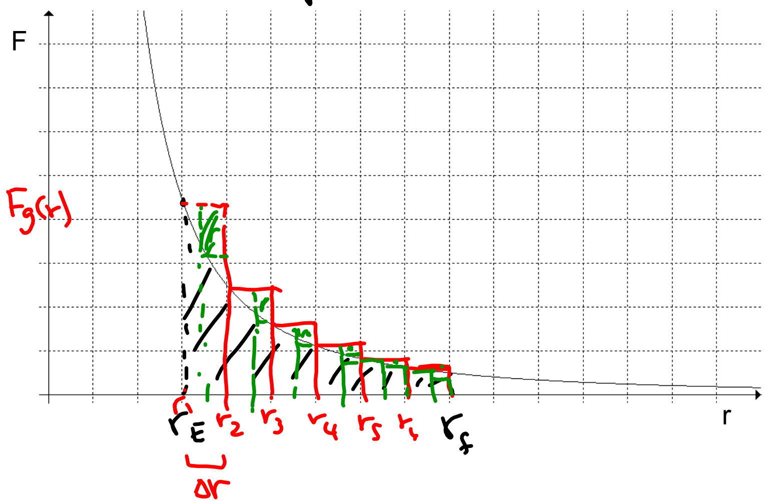
Spring



$$F_g = \frac{GMm}{r^2}$$

How do we calculate the area of that?

Calculus - specifically integration



$$\text{Area} \approx \sum_{i=1}^{6} F_g(r_i)\Delta r$$

$$= \lim_{\Delta r \to 0} \sum_{i=1}^{n} F_g(r_i)\Delta r$$

$$\frac{d}{dr}\int F_g(r)\,dr = F_g(r)$$

$$= \int_{r_E}^{r_f} F_g(r)\,dr$$ ← (like a teeny teeny tiny Δr.)

integral → from r_E to r_f of $F_g(r)\,dr$

$$\frac{d}{dr}\,? = \frac{GMm}{r^2}$$

$$= GMm\, r^{-2}$$ (GMm: const.)

$$\frac{d}{dr}\left(\frac{GMm}{r}\right) = \frac{GMm}{r^2}$$

$$\frac{d}{dr}\,\underbrace{?} = r^{-2}$$

$$\frac{d}{dr}\,-r^{-1} = +1\,r^{-2} = r^{-2}$$

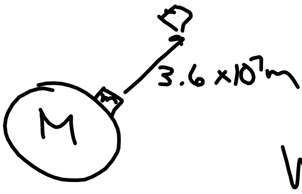
$$\text{Area} = -\frac{GMm}{r_f} - \left(-\frac{GMm}{r_E}\right) = GMm\left(\frac{1}{r_E} - \frac{1}{r_f}\right)$$

Potential Energy ($r \gg r_E$ or any other planet/star)

Define $PE(r) = -\frac{GMm}{r}$ $PE(\infty) = 0$

remember in physics ∞ just means really big.

How much energy does it take to lift a satellite of mass 1000 kg to an altitude of 3.6×10^7 m?



$r_1 = 6.4 \times 10^6$ m

$r_2 = 3.6 \times 10^7 + 6.4 \times 10^6$ m $= 4.24 \times 10^7$ m

$W = \Delta PE = PE_f - PE_i$

$= \frac{-GMm}{r_2} - \left(-\frac{GMm}{r_1}\right)$

$W = -(6.67 \times 10^{-11})(5.98 \times 10^{24})(1000\text{ kg})\left(\frac{1}{4.24 \times 10^7} - \frac{1}{3.6 \times 10^6}\right)$

$= -4.0 \times 10^{17}(-2.5 \times 10^{-7})$

$= 1.0 \times 10^{11}$ J

Aside if g were constant (and it's NOT)

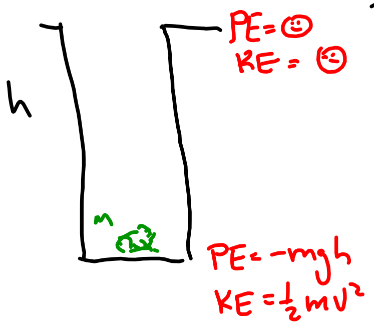
$W = mgh$

$= 1000 \cdot 10 \, (3.6 \times 10^7)$

$= 3.6 \times 10^{12}$ J

Escape Velocity

Example: The frog in the well. . .



How fast does froggy have to jump to get out of the well?

If he jumps <u>just</u> fast enough to get out, KE at top is zero.

So to jump out, by cons of E

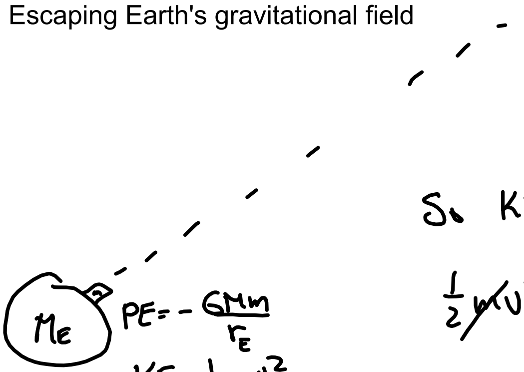
$$(KE + PE)_{bottom} = (KE + PE)_{top} = ☺$$

So $KE_{bottom} = +mgh = \frac{1}{2}mv^2$

$$V_{esc} = \sqrt{2gh}$$

Escape Velocity

Example: Escaping Earth's gravitational field



$PE = 0$, $KE = 0$ at $r = $ "∞"

$PE = -\frac{GMm}{r_E}$

$KE = \frac{1}{2}mv^2$

So $KE = -PE = \frac{GMm}{r_E}$

$\frac{1}{2}\cancel{m}v^2 = \frac{GM\cancel{m}}{r_E}$

$v_{esc} = \sqrt{\frac{2GM}{r_E}} = \sqrt{\frac{2(6.67\times10^{-11})(5.98\times10^{24})}{6.38\times10^{6}}}$

$= 1.12\times10^{4}$ m/s

or $= 11.2$ km/s

1. What is the escape velocity from the surface of Jupiter?

2. The moon has a mass of 7.35×10^{22} kg and orbits the Earth at a radius of 3.84×10^{8} m.
   a) What is the gravitational potential energy of the Moon-Earth system?
   b) What is the Moon's kinetic energy and speed in circular orbit?
   c) What is the Moon's binding energy to Earth?

3. What is the total energy needed to place a 2000 kg satellite into circular orbit around Earth at an altitude of 5.0×10^{2} km?