$r_E = 6.38\times10^6 m$

$$F_g = \frac{GMm}{r^2}$$

$\frac{4\times10^{17}}{50\times10^{16}}$ $\frac{4}{5}\times10^0 = 0.8N$

$$= \frac{(6.67\times10^{-11})(5.98\times10^{24} kg)(1012 kg)}{(7.11\times10^8 m)^2}$$

$= 0.79 N$

$g = \frac{GM}{r^2}$

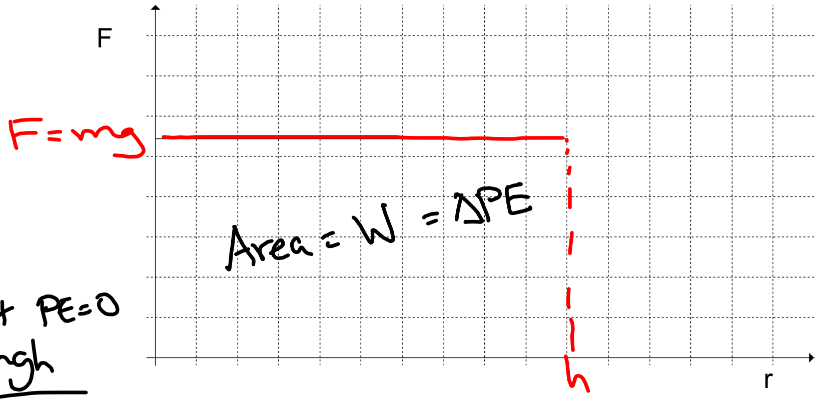
Gravitational Potential Energy

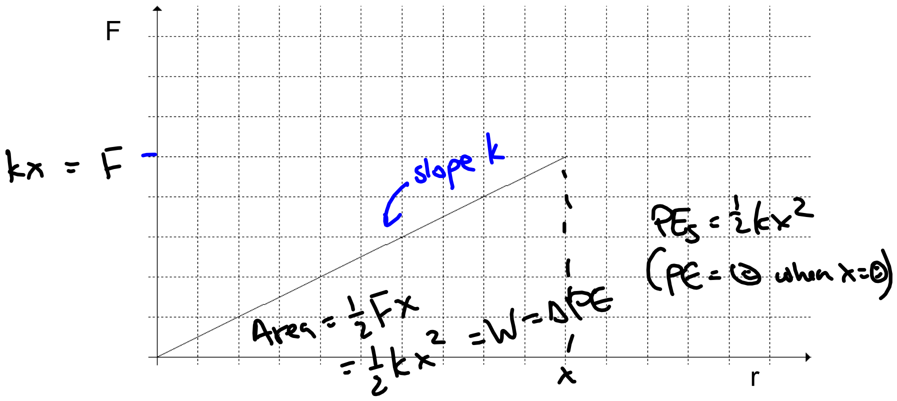
Recall W=Fd

$W = Fd$

$\Delta PE = mgh$

If starting point PE=0 then $PE = mgh$

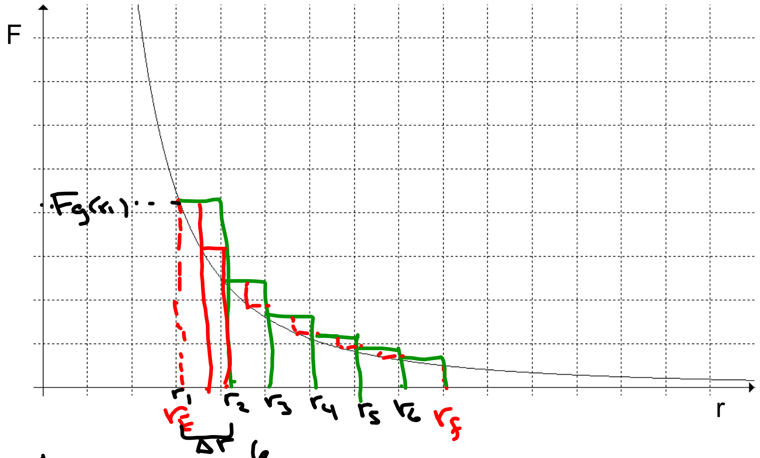




If $r > r_E$ $\qquad F_g = \frac{GMm}{r^2}$

How do we find the area of that thing?

Calculus.



$$\text{Area} \approx \sum_{i=1}^{6} F_g(r_i)\Delta r$$

$$= \lim_{\Delta r \to 0} \sum_{i=1}^{n} F_g(r_i)\Delta r$$

$$= \int_{r_E}^{r_f} F_g(r)\,dr$$

integral (sum) between r_E and r_f.

height of ▭

← teeny teeny tiny Δr

$$\frac{d}{dr}\int F_g(r)\,dr = F_g(r)$$

$$\frac{d}{dr}\,\underline{\;?\;} = \frac{GMm}{r^2}$$

$$\frac{d}{dr}\left(-\frac{GMm}{r}\right) = GMm\,r^{-2}$$

$$\frac{d}{dr}\,\underline{\;?\;} = r^{-2}$$

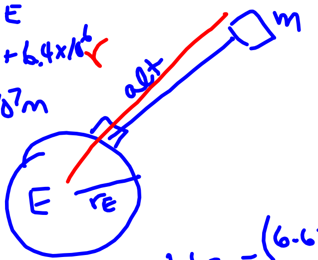
$$? = -r^{-1}$$

$$\int_{r_E}^{r_f} \frac{GMm}{r^2}\,dr = -\frac{GMm}{r_f} - \left(-\frac{GMm}{r_E}\right) = -GMm\left(\frac{1}{r_f} - \frac{1}{r_E}\right)$$

Define $PE_g(r) = -\frac{GMm}{r}$ defined by $PE(\infty) = 0$ ☺

How much energy does it take to lift a satellite ($m = 1000\,kg$) to an altitude of $3.6 \times 10^7\,m$ above earth's surface?

$r = alt + r_E$
$= 3.6\times10^7 + 6.4\times10^6$
$= 4.24\times10^7\,m$



$W = \Delta PE$
$= PE_f - PE_i$
$= -\frac{GMm}{r} - \left(-\frac{GMm}{r_E}\right) = -GMm\left(\frac{1}{r} - \frac{1}{r_E}\right)$

$W = -(6.67\times10^{-11})(5.98\times10^{24})(1000)\left[\frac{1}{4.24\times10^7} - \frac{1}{6.4\times10^6}\right]$

$= + 4.0\times10^{17}\left[+1.3\times10^{-7}\right]$

$= 5.2\times10^{10}\,J$

What if g were constant (and it is NOT)

then $\Delta PE = mgh$
$= (1000)(10)(3.6\times10^7)$

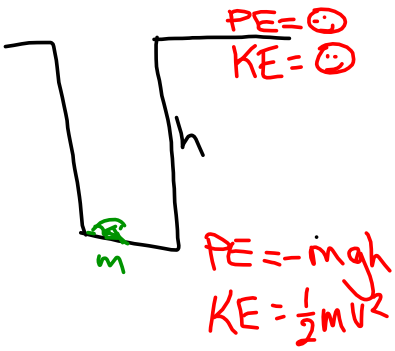
$0.2 \leq g \leq 9.8$ (a_g at r ... a_g at r_E)

too big $= 3.6\times10^{11}\,J$

too small $\Rightarrow 7.2\times10^9\,J$

Escape Velocity

Example: The frog in the well. . .



How fast does froggy need to jump to escape the well?

By cons of E

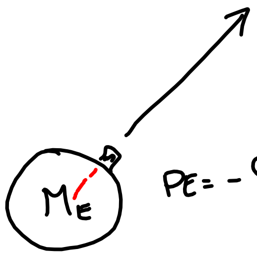
$$(KE + PE)_{bot} = (KE + PE)_{top}$$

$$\frac{KE}{m} = \frac{1}{2}\cancel{m}v^2 = \cancel{m}gh$$

$$v_{esc} = \sqrt{2gh}$$

Escape Velocity

Example: Escaping Earth's gravitational field



$r = \infty$ $\quad PE = 0, \; KE = 0 \; \Big] \; E_T = 0$

$PE = -\dfrac{GM_E m}{r_E}, \quad KE = \frac{1}{2}mv^2 \; \Big] \; E_T = 0$

$$\frac{1}{2}m v_{esc}^2 = \frac{GM_E m}{r_E}$$

$$v_{esc} = \sqrt{\frac{2GM_E}{r_E}}$$

$$= \sqrt{\frac{2(6.67\times10^{-11})(5.98\times10^{24})}{6.38\times10^{6}}}$$

$$= 1.12\times10^{4}\ \text{m/s}$$

$$= 11.2\ \text{km/s}$$

1. What is the escape velocity from the surface of Jupiter?

2. The moon has a mass of 7.35×10^{22} kg and orbits the Earth at a radius of 3.84×10^{8} m.
    a) What is the gravitational potential energy of the Moon-Earth system?
    b) What is the Moon's kinetic energy and speed in circular orbit?
    c) What is the Moon's binding energy to Earth?

3. What is the total energy needed to place a 2000 kg satellite into circular orbit around Earth at an altitude of 5.0×10^{2} km?