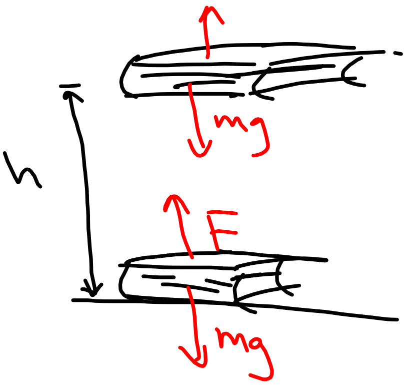
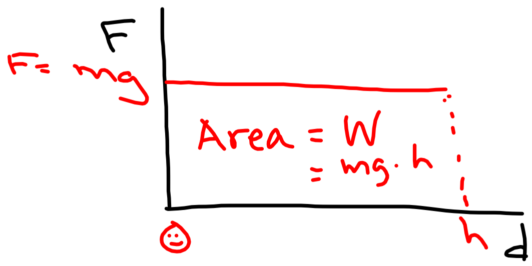
Gravitational Potential Energy



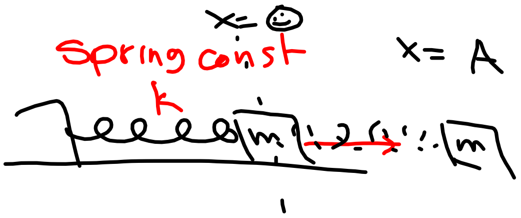
$$PE = mgh$$

true for $h = \Delta r \ll r_E$

$$W = F_{\parallel} d = F \cdot h = mgh$$



$$\Delta PE = W$$
$$= \text{Area}$$



$$W = \frac{1}{2} bh$$
$$= \frac{1}{2} FA$$
$$= \frac{1}{2}(kA)A$$
$$= \frac{1}{2} kA^2$$
$$\Delta PE = \frac{1}{2} kA^2$$

What if $r >> r_E$?

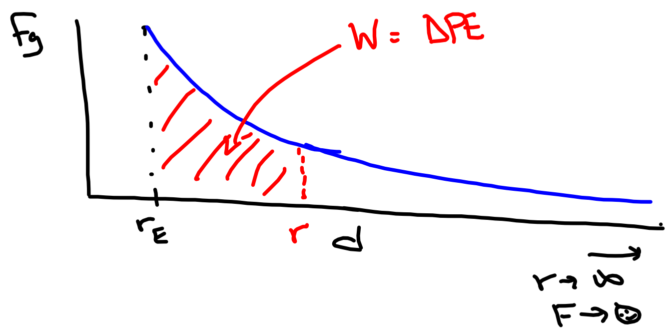
Recall:

$$F_g = \frac{GMm}{r^2}$$

$M = m_{earth}$

Define $PE(\infty) = ☺$

$PE(r < \infty) = -$

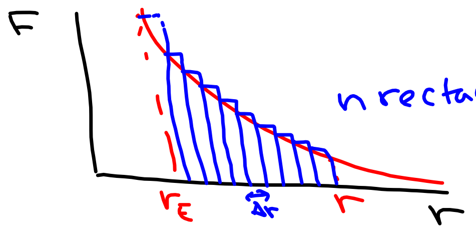


Math $\infty = \infty$

Physics ∞ = really big compared to.

(no measurable difference)

So - how do we find the area under that?



n rectangles

$\text{Area} \approx \sum \text{Area}_{\square}$

$= \sum F_g(r)\,\Delta r$

$= GMm \sum_{i=1}^{n} \frac{1}{r_i^2}$

$r_1 = r_E$

$r_2 = r_E + \Delta r$

$\vdots$

$r_n = r_E + (n-1)\Delta r$

More $\square$s is better!!

$\lim \Delta r \to 0$

Area = Area $\square$s

like a teeny tiny Δr.

$$\int F_g(r)\,dr = \lim_{\Delta r \to 0} \sum F_g(r)\,\Delta r$$

sum of infinite # of things

sum

$$PE(r) = \int \frac{GMm}{r^2}\,dr$$

$$\frac{d}{dr}\int F_g(r)\,dr = F_g(r)$$

$$\frac{d}{dr}\,? = r^{-2} \qquad \frac{d}{dr}PE(r) = GMm\,r^{-2}$$

$? = -r^{-1}$

$$\frac{d}{dr}\left(-r^{-1}\right) = +1\,r^{-1-1} = r^{-2}$$

$$PE(r) = -\frac{GMm}{r} + C$$

because it's a choice for PE (relative to something)

We choose $C = 0$

Which means $PE = 0$ when $r = \infty$

Define $PE = 0$ at $r = \infty$

$$\Rightarrow \boxed{PE(r) = -\frac{GMm}{r}}$$

$$\text{Area} = \Delta PE = PE(r) - PE(r_E)$$

$$= -GMm\left(\frac{1}{r} - \frac{1}{r_E}\right)$$

$$= GMm\left(\frac{1}{r_E} - \frac{1}{r}\right)$$

OR in general

$$A = -GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

How much energy does it take to put a satellite of mass 1000 kg into orbit at a radius of $3r_E$?

$M_e = 5.97\times10^{24}$ kg

$r_E = 6.38\times10^{6}$ m

$E = \Delta KE + \Delta PE$

$\downarrow$ KE_{orbit} - ☺ "ish"

$\downarrow$ $PE(3r_E) - PE(r_E)$

$$KE_{orbit} = \frac{1}{2}\frac{mv^2}{\cancel{r}} = \frac{1}{2}\frac{GMm}{r^{\cancel{2}}} = \frac{1}{2}\underbrace{\frac{GMm}{r}}_{-PE_{orbit}}$$

$$E = \underbrace{\frac{1}{2}\frac{GMm}{3r_E}}_{KE_{orbit}} + \left(-\frac{GMm}{3r_E} + \frac{GMm}{r_E}\right)$$

$$E = \frac{GMm - 2GMm + 6GMm}{6r_E}$$

On Earth

$PE = -\frac{GMm}{r}$

$$= \frac{GMm}{6r_E}(1 - 2 + 6)$$

$$= \frac{5}{6}\frac{GMm}{r_E}$$

$$= \frac{5}{6}\frac{(6.67\times10^{-11})(5.97\times10^{24})(1000\text{ kg})}{6.38\times10^{6}}$$

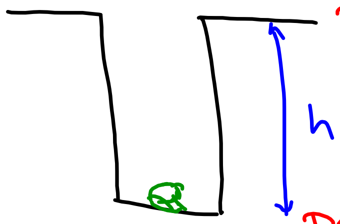
$$= 5.2\times10^{10}\text{ J} = 52\text{ GJ}$$

Escape Velocity:

Frog in a well

assume $a_y = g$

How fast does froggy have to jump to escape the well?



PE = ☺
KE = ☺

PE = $-mgh$
KE = $\frac{1}{2}mv_i^2$

By cons of E

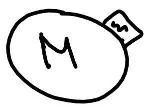
$$\frac{1}{2}\cancel{m}v_i^2 = \cancel{m}gh$$

$$v_i = \sqrt{2gh}$$

$$V_{esc} = \sqrt{2gh}$$

Escape Velocity

How fast do we need to launch a satellite to escape Earth's gravitational pull?



$PE_i = -\frac{GMm}{r_E}$

$PE_f =$ ☺

$KE_i = \frac{1}{2}mv_i^2$

$KE_f =$ ☺

By cons of E

$PE_i + KE_i = PE_f + KE_f$

$=$ ☺

$\frac{1}{2}\cancel{m}v_{esc}^2 = \frac{GM\cancel{m}}{r_E}$

$v_{esc} = \sqrt{\frac{2GM}{r_E}}$

$= \sqrt{\frac{2(6.67\times10^{-11})(5.97\times10^{24})}{6.38\times10^{6}}}$

$= 1.12\times10^{4}$

$= 11.2\ \text{km/s}$

rotation of earth ... v at equator

$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(6.38\times10^6)}{(86400s)^2} =$

$= 0.03\ m/s^2$ small

around sun $a = \frac{v^2}{r} = \frac{(29.8\times10^3\ \frac{m}{s})^2}{1.5\times10^{11}\ m} = \frac{9\times10^8}{1.5\times10^{11}} = 6\times10^{-3}\ \frac{m}{s^2}$

negligible

Gravitational Field

Define the gravitational field

acceleration due to gravity

$$\vec{a}_g = \vec{g} = \frac{\vec{F}_g}{m} = -\frac{\frac{GM\cancel{m}}{r^2}}{\cancel{m}}\hat{r} = -\frac{GM}{r^2}\hat{r}$$