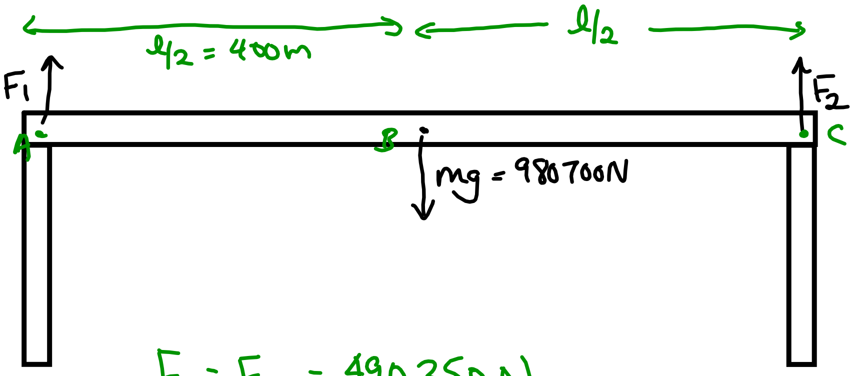
Homework Questions

Static Equilibrium

Outcomes: Students will be able to use vector analysis to solve for forces in static equilibrium cases involving torques.

Textbook: Section 10.3

Consider a bridge whose platform (m=100 000 kg) is uniform and is 800 m long, supported by a pier on each end of the bridge. What is the force that each pier applies to keep the bridge up?



$F_1 = F_2 = 490\,350\,N$

$F_1 + F_2 = mg$ ← $\Sigma F = 0$

$\Sigma \tau_B = 0$

$\Sigma \tau_{ccw} = \Sigma \tau_{ccw}$

$F_1 \cdot \frac{l}{2} = F_2 \cdot \frac{l}{2}$

$F_1 = F_2$

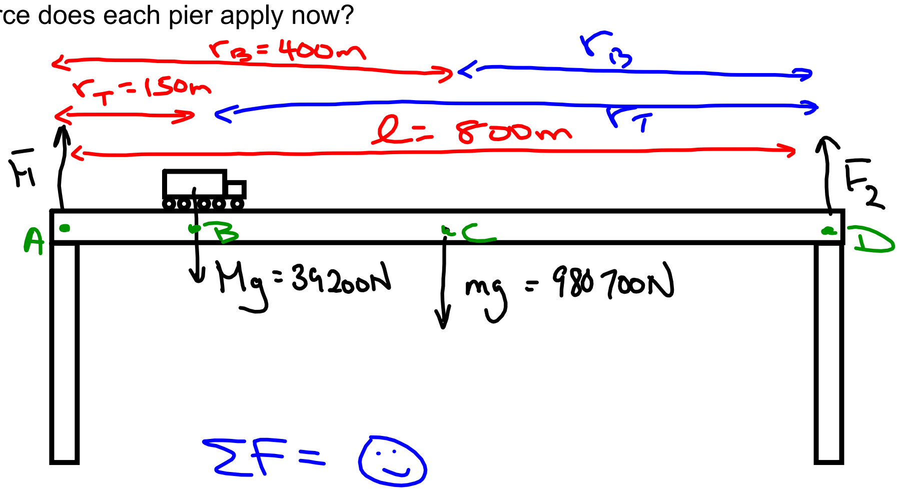
$F_1 + F_2 = 980\,700N$

$F_1 + F_1 = "$

$2F_1 = "$

$F_1 = 490\,350N = F_2$

Now consider the same bridge with a truck of mass 4000 kg a distance of 150 m from one end. How much force does each pier apply now?



$\Sigma F = 😊$

$F_1 + F_2 = mg + Mg = 1019900N$

But $\Sigma \tau = 😊$ as well. Around any and every point in space?

Choose A. $\Sigma \tau_A = 😊$

$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$

$Mg \cdot r_T + mg \cdot r_B = F_2 \cdot \ell$

$\frac{39200N \cdot 150m + 980700N \cdot 400m}{800m} = F_2 \cdot \cancel{800m}$

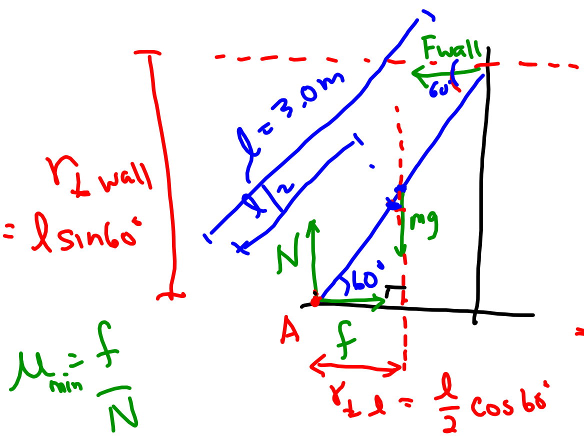
$F_2 = 497700N$

but $F_1 + F_2 = 1019900N$

So $F_1 = 522200N$

Around D $\Sigma \tau_D = 😊$

A ladder (ℓ = 3.0 m, m=10 kg) is leaned against a wall at an angle of 60°. If we assume that the friction with the wall is negligible, what coefficient of friction is required with the ground to keep it from slipping?



$\Sigma F = ☺$

$\Sigma F_x = ☺$ $\Sigma F_y = ☺$

$F_{wall} = f$ $N = mg = 98N$

$\Sigma \tau_A = ☺$

Recall: $r_\perp$ is the $\perp$ dist from point of rotation to the line of force

$\Rightarrow mg \cdot r_{\perp \ell} = F_{wall} \cdot r_{\perp wall}$

$98 \cdot 1.5\cos 60° = F_{wall} \cdot 3.0\sin 60°$

$F_{wall} = 28.3N = f$

$\mu = \frac{f}{N} = \frac{28.3N}{98N} = 0.29$

$\tau = rF\sin\theta$ ← angle between them

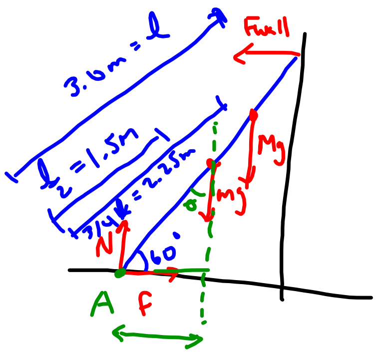
$\frac{\ell}{2} \cdot mg \sin 30$

$mg\, r_{\perp \ell} = F_{wall}\, r_{\perp wall}$

$mg \cdot \frac{r_{\perp \ell}}{r_{\perp wall}} = F_{wall} = f$

$\mu_{min} = \frac{f}{N} = \frac{mg \cdot \frac{r_{\perp \ell}}{r_{\perp wall}}}{mg}$

A person (m=70 kg) climbs 3/4 of the way up the ladder. What coefficient of friction is required with the ground now?



$\Sigma F = 0$

$N = mg + Mg$

$= 785N$

$f = F_{wall}$

$\Sigma \tau_A = 0$

$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$

$mg \cdot r_{\perp l} + Mg \cdot r_{\perp p} = F_{wall}\, r_{\perp wall}$

$$\frac{98.1 \cdot 1.5m \cos 60° + 686.5 \cdot 2.25 \cos 60°}{3.0 \sin 60°} = F_{wall} \cdot \cancel{3.0 \sin 60°}$$

$F_{wall} = 326N = f$

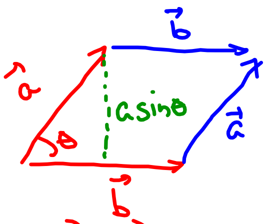
$$\mu = \frac{f}{N} = \frac{326N}{785N} = 0.41$$

Direction of Torque and Cross products

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{a} \times \vec{b}$ ⊗ ← into board

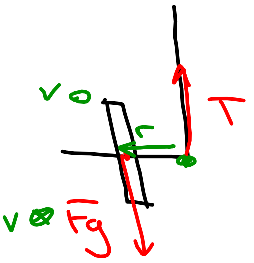
$\vec{b} \times \vec{a}$ ⊙ ← out of board



$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$ = Area of llgm.

So, $\vec{a} \times \vec{b}$ represents the area mapped out by $\vec{a}$ and $\vec{b}$, but what is the direction? Perpendicular to the surface, but again... which way?

RH rule
- Fingers in direction of $\vec{a}$
- wrap them toward $\vec{b}$
- thumb points in direction of $\vec{a} \times \vec{b}$



Homework:
Sheet #2, 6-10