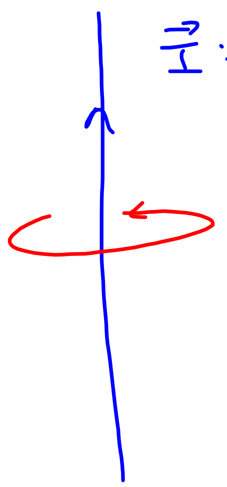
Magnetic Fields and Wires

Magnetic field from a "long" wire

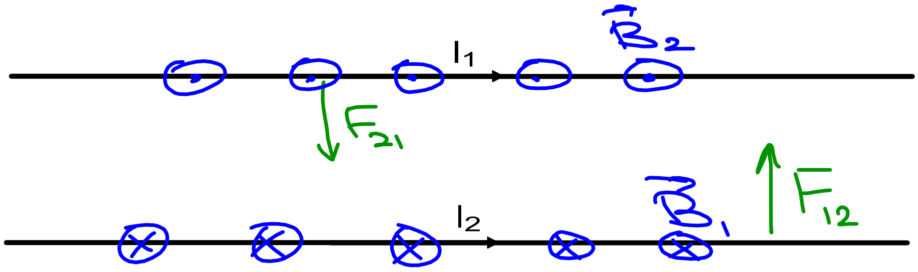
Mathematically, "long" = infinitely long
Physically, "long" = long compared to other values of interest



$$\vec{I} = I\hat{z}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\theta}$$

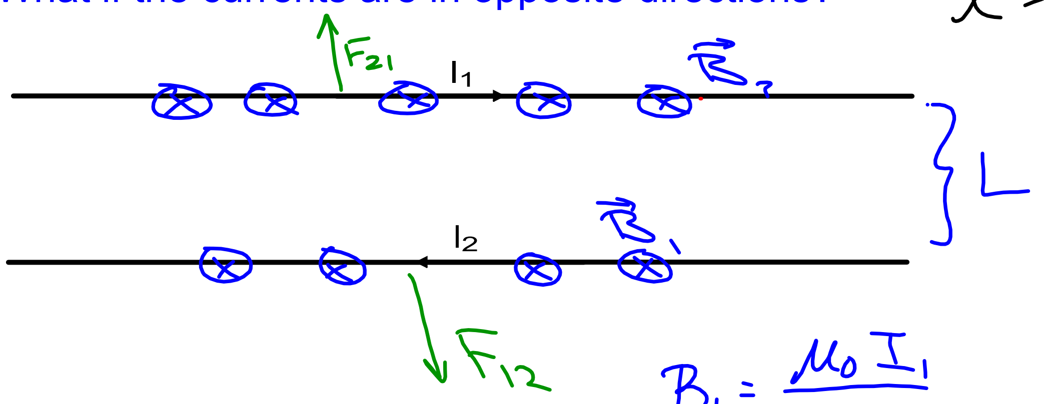
Two long parallel wires. . .



Since each wire creates a magnetic field, do the two magnetic fields interact?
If so, what force does one apply to the other (note Newton's 3rd law would state that F_{12}=-F_{21})?

What if the currents are in opposite directions?

$\ell >> L$



$$B_1 = \frac{\mu_0 I_1}{2\pi L}$$

Force experienced by wire

$$F = \ell I B$$

$$F_{12} = \ell I_2 B_1$$

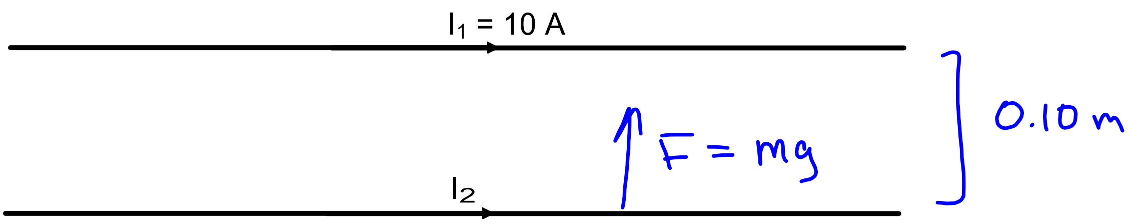
$$F_{12} = \ell \frac{\mu_0 I_1 I_2}{2\pi L}$$

$$F_{21} = \ell I_1 B_2 = \ell I_1 \left(\frac{\mu_0 I_2}{2\pi L}\right) = \ell \frac{\mu_0 I_1 I_2}{2\pi L}$$

ℓ = length of wire's mutually experiencing the field.

$$\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi L}.$$

Levitating wires (the bottom one). . .



Assume the bottom wire is suspended 0.10 m below the upper wire, by the upper wire's magnetic field. Assume the upper wire is carrying a sizeable current of 10 A. If the mass per unit length, μ = 50 g/m , what must be the current in the bottom wire.

$$\mu = \frac{m}{l} \qquad \frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi L}$$

$$\left(\frac{m}{l}\right)g = \frac{mg}{l} = \frac{\mu_0 I_1 I_2}{2\pi L}$$

$$50\times10^{-3}\,\frac{kg}{m}\cdot 9.807\,m/s^2 = \frac{\left(4\pi\times10^{-7}\,\frac{T}{A\cdot m}\right)(10\,A)\,I_2}{2\pi\,(0.10\,m)}$$

$$0.49\,\frac{N}{m} = 2.0\times10^{-5}\,\frac{T}{m^2}\,I_2$$

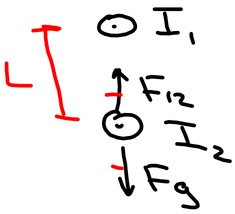
$$24500\,A = I_2 \quad !!$$

$$I_1 I_2 = 50\cdot 24500$$
$$= 1225000$$

for min I_1/I_2

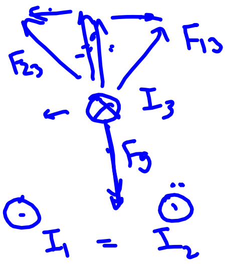
$$I_1 I_2 = I^2 = 122500$$
$$I \approx 1.1125\times10^3$$
$$= \underline{1100\,A} \quad !!$$

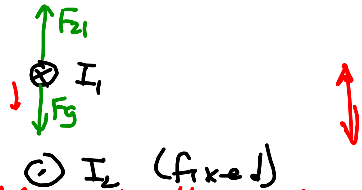
Stable vs. Unstable Equilibria (and Saddle Points)



unstable

- If L increases F_{12} decreases
  wire falls
- If L decreases F_{12} increases
  wire is pulled up

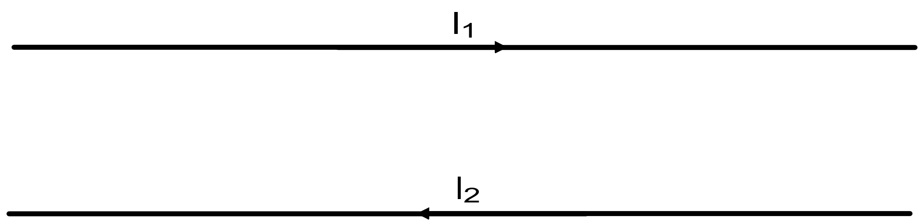




Stable vertically only

- If L decreases
  F_{21} increases
  wire pushed upward
- If L increases
  F_{21} decreases
  wire falls

Saddle point – stable in one dimension ↕
unstable in another ⟷



If the upper wire is suspended by repulsion from the lower wire, is it stable or unstable (up and down)? What about side to side? If unstable, what could you do to make it stable?