Cylindrical Coordinate Systems and Ampere's Law

n-hat → Unit vectors

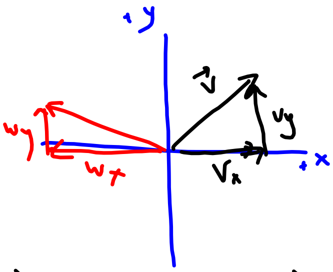
$\hat{n}$ = a vector of length 1 in the increasing n-direction

Cartesian Coordinates and Unit Vectors

2D

$\vec{V} = \vec{V}_x + \vec{V}_y$

$= V_x \hat{x} + V_y \hat{y}$



$\hat{x}$ = vector of length 1 in the increasing x-direction

$\vec{V} = V_x \hat{x} + V_y \hat{y}$

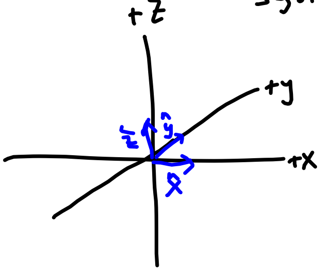
$\vec{W} = -W_x \hat{x} + W_y \hat{y}$

$\vec{V} + \vec{W} = (V_x - W_x)\hat{x} + (V_y + W_y)\hat{y}$

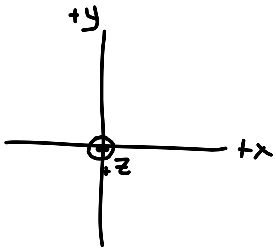
orthogonal ⇒ perpendicular

3D

RH - coordinate system



$\vec{V} = V_x \hat{x} + V_y \hat{y} + V_z \hat{z}$



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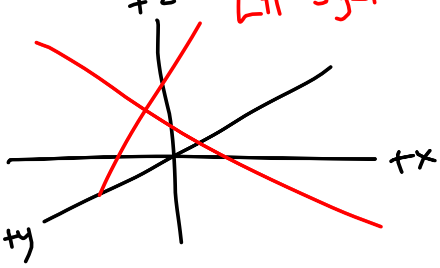
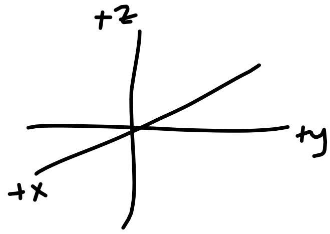


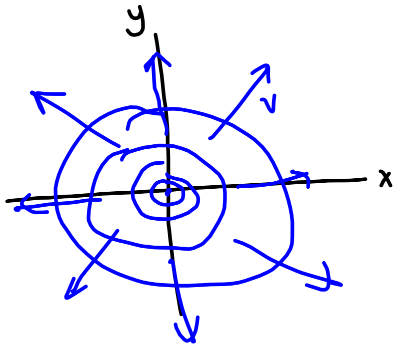
LH system

RH



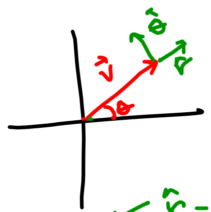
useful for circular or radial motions

2D Polar Coordinate System and Unit Vectors



$\hat{r}$

$\hat{\theta}$



$\vec{v} = v\hat{r}$ ← $\hat{r} = \hat{r}(\theta)$

$\hat{\theta} = \hat{\theta}(\theta)$

$\hat{r}$ = radially outward

$-\hat{r}$ = radially inward

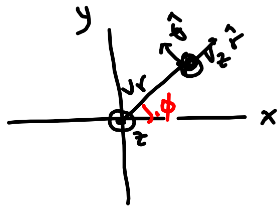
$\hat{\theta}$ = counter clockwise

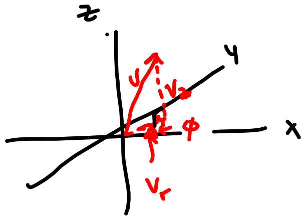
$-\hat{\theta}$ = clockwise

Cylindrical Coordinate Systems and Unit Vectors

x-y plane we use a polar system

z direction we keep z

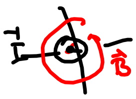




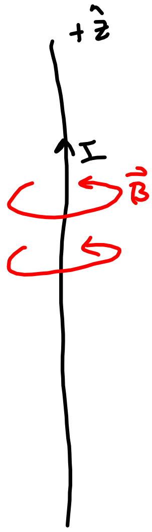
$\langle r, \phi, z \rangle$

Applying Cylindrical Coordinates Systems to a Current Carrying Wire

$\vec{V} = V_x \hat{x} + V_y \hat{y} + V_z \hat{z}$



$\vec{B} = B_\phi \hat{\phi}$



$\vec{I} = I\hat{z}$

$\vec{B} = B_\phi \hat{\phi}$

$B_r = 0$

$B_z = 0$

$\vec{B} = \cancel{B_r \hat{r}} + B_\phi \hat{\phi} + \cancel{B_z \hat{z}}$

Ampere's Law

Ampere's Law describes the relationship between any current carrying wire and the magnetic field.

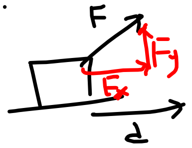
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{in}$$

where I_{in} is the current that passes through the closed path.

Integral over a closed path

dot product of $\vec{B}$ and $d\vec{l}$

$d\vec{l}$ is a teeny-tiny l vector.



$W = F_{\parallel} d$

$= F_x d$

$\Downarrow$

non-calculus approximation

$$\sum B_{\parallel} \Delta l = \mu_0 I_{in}$$

$\vec{F} = F_x \hat{x} + F_y \hat{y} + F_z \hat{z}$

$\vec{d} = d\hat{x}$

$\hat{x} \cdot \hat{x} = 1 = \hat{y} \cdot \hat{y} = \hat{z} \cdot \hat{z}$

$\hat{x} \cdot \hat{y} = \hat{y} \cdot \hat{x} = \hat{x} \cdot \hat{z} = \hat{z} \cdot \hat{x}$

$= \hat{y} \cdot \hat{z} = \hat{z} \cdot \hat{y} = 0$

Closed loop is a mathematical construct.

<u>NOT</u> a physical loop

Applying Ampere's Law to a long current carrying wire

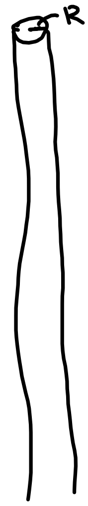
$\ell >> r$

By symmetry

B is not a function of z

B is not a function of θ

$\underline{\vec{B} = \vec{B}(r)}$



$\vec{I} = I\hat{z}$

$B_r = 0$ by Biot-Savart

We are not doing this

Start with

$\vec{B} = B_r\hat{r} + B_\phi\hat{\phi} + B_z\hat{z}$

$\vec{B} = \vec{B}(r,\phi,z)$

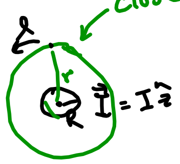
$B_r = B_r(r,\phi,z)$

$B_\phi = B_\phi(r,\phi,z)$

$B_z = B_z(r,\phi,z)$

$\vec{B} = \overset{0}{\cancel{B_r(r)}}\hat{r} + B_\phi(r)\hat{\phi} + B_z(r)\hat{z}$

B_ϕ outside the wire $(r > R)$



current passing through closed loop

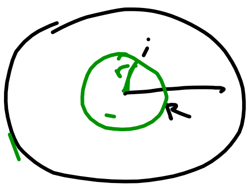
$$\sum B_{\parallel} \Delta l = \mu_0 I_{in}$$

$$B_\phi(r)(2\pi r) = \mu_0 I$$

$$B_\phi(r) = \frac{\mu_0 I}{2\pi r}$$

$$\vec{B}_\phi(r) = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

B_ϕ inside the wire $(r < R)$



$\vec{I} = I\hat{z}$ uniformly distributed through wire

$$\sum B_{\parallel} \Delta l = \mu_0 I_{in}$$

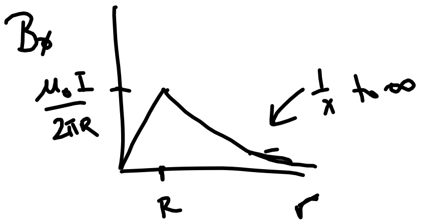
$$B_\phi(r)(2\pi r) = \mu_0\left(I\frac{r^2}{R^2}\right)$$

$\text{Area}_{o} = \pi r^2$

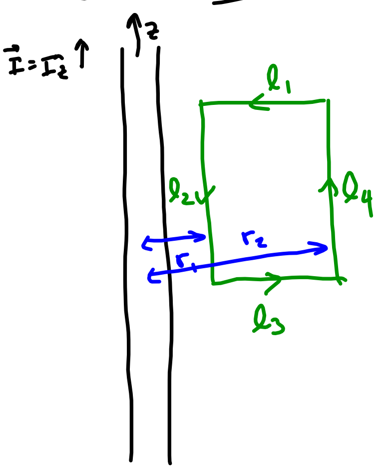
$\text{Area wire} = \pi R^2$

$$I_{in} = I \cdot \frac{\pi r^2}{\pi R^2} = \frac{Ir^2}{R^2}$$

$$B_\phi = \frac{\mu_0 I r}{2\pi R^2} \quad \text{at } r = R$$



B_z outside the wire $\quad B_r = 0 \quad\quad l_2 = l_4$



$$\sum B_\parallel l = \mu_0 \cancelto{0}{I_{in}}$$

-r direction

$$\cancelto{0}{B_r(-l_1)} + B_z(r_1)(-l_2) + \cancelto{0}{B_r(l_3)} + B_z(r_2) l_4 = 0$$

$$B_z(r_2)(\cancel{l_4}) = B_z(r_1)(\cancel{l_2})$$

$$B_z(r_1) = B_z(r_2) \quad \forall\, r > R$$

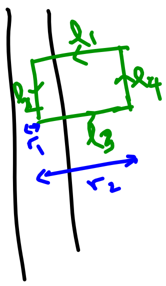
B_z outside wire is constant

But $B_z(\infty) = 0$

$\therefore B_z = 0$ outside wire

B_z inside

$r_1 < R$
$r_2 > R$



$$\sum B_\parallel \Delta l = \mu_0 \cancelto{0}{I_{in}}$$

$$-\cancelto{0}{B_r l_1} - B_z(r_1) l_2 + \cancelto{0}{B_r l_3} + \cancelto{0}{B_z(r_2)} l_4 = 0$$

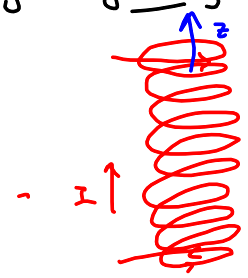
$\therefore B_z(r_1) = 0 \quad$ for all $r < R$

long → ∞ ≫ ℓ

And a Solenoid

N loops per unit length

Again symmetry

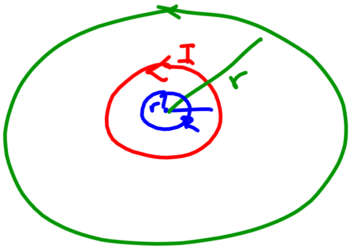


$B_z = B_z(r)$

$B_\phi = B_\phi(r)$

$B_r = $ ☹ Biot Savart

B_ϕ outside



$\Sigma B_{\parallel} \Delta \ell = \mu_0 I_{in}$

$B_\phi(r)\, 2\pi r = \mu_0 I$

$B_\phi(r) = \dfrac{\mu_0 I}{2\pi r}$

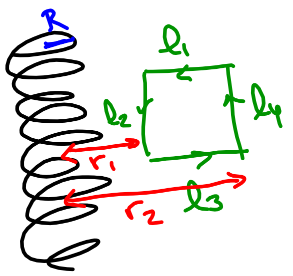
B_ϕ inside

$\Sigma B_{\parallel} \Delta \ell = \mu \cancel{I_{in}}$ ☺

$B_\phi(r)\, 2\pi r = $ ☹

$B_\phi = $ ☺

B_z outside $r_1, r_2 > R$



$$\Sigma B_\parallel \Delta l = \cancel{\mu_0 I_{in}}$$

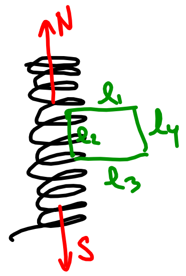
$$-B_r l_1 - B_z(r_1) + B_r l_3 + B_z(r_2) l_4 = ☺$$

Same as wire

$$B_z = ☺ \quad \forall r > R$$

B_z inside

$I_{in} = I N l_2$



$$\Sigma B_\parallel \Delta l = \mu_0 I_{in}$$

$$-\cancel{B_r l_1} - B_z(r_1)\cancel{l_2} + \cancel{B_r l_3} + \cancel{B_z(r_2)} l_4 = \mu_0 I N \cancel{l_2}$$

$$B_z(r_1) = \mu_0 I N$$

$$B_z = \mu_0 I N$$