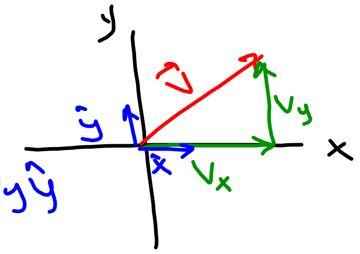
# Cylindrical Coordinate Systems and Ampere's Law

Cartesian Coordinates and Unit Vectors

2D



$\vec{W} = W_x\hat{x} + W_y\hat{y}$

$\vec{V} + \vec{W} = (V_x + W_x)\hat{x} + (V_y + W_y)\hat{y}$

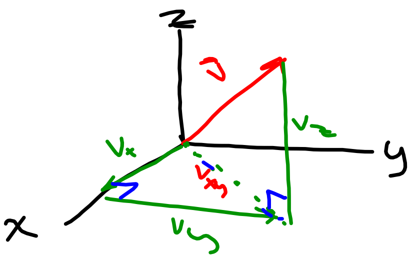
$\vec{V} = \vec{V}_x + \vec{V}_y$

$= V_x\hat{x} + V_y\hat{y}$

$|\vec{V}|^2 = V_x^2 + V_y^2 \qquad |\vec{V}|^2 = \vec{V}\cdot\vec{V}$

$\hat{n}$ = a vector of length 1 in the increasing $n$ direction

3D



RH.
rotate x into y
thumb points in z

$\hat{x} \times \hat{y} = \hat{z}$

$\hat{y} \times \hat{x} = -\hat{z}$

$W = F_{\parallel} d$

$= \vec{F}\cdot\vec{d}$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{V} = V_x\hat{x} + V_y\hat{y} + V_z\hat{z}$

$|\vec{V}|^2 = \vec{V}\cdot\vec{V}$

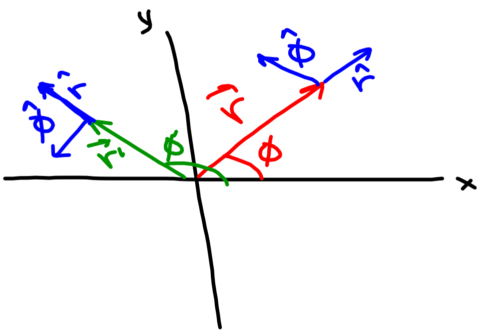
$= V_x^2 + V_y^2 + V_z^2$

$V_{xy}^2 = V_x^2 + V_y^2$

$V^2 = V_{xy}^2 + V_z^2$

$= V_x^2 + V_y^2 + V_z^2$

## 2D Polar Coordinate System and Unit Vectors



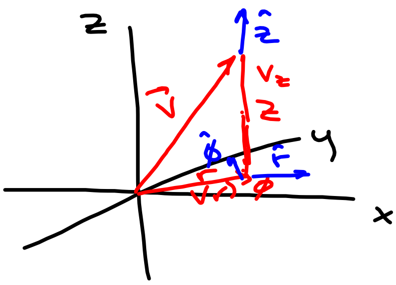

$$\vec{r} = r\,\hat{r}$$

$$\vec{r}\,' = r'\,\hat{r}$$

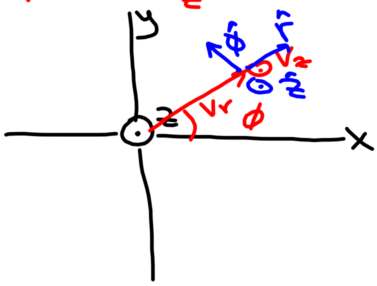
$$\hat{r} = \hat{r}(\phi)$$

$$\hat{\phi} = \hat{\phi}(\phi)$$

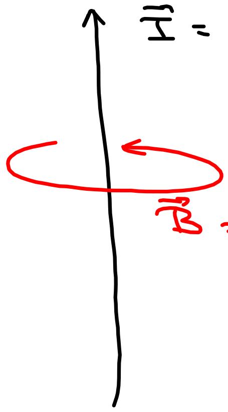
Cylindrical Coordinate Systems and Unit Vectors



$$\vec{V} = V_r\,\hat{r} + V_z\,\hat{z}$$



Applying Cylindrical Coordinate Systems to a Current Carrying Wire



$\vec{I} = I\hat{z}$

$$\vec{B} = \vec{B}_\phi = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

## Ampere's Law

Ampere's Law describes the relationship between any current carrying wire and the magnetic field.

magnetic field

$$\oint \vec{B} \cdot \underbrace{d\vec{l}}_{\text{teeny, tiny } \Delta\vec{l}} = \mu_0 \underbrace{I_{in}}$$

the sum over the closed path, $l$.

current that passes through an arbitrary closed path, $l$.

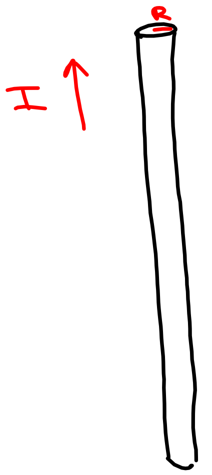
$\vec{B} \cdot d\vec{l}$ is the product of the parallel component of $\vec{B}$ with $d\vec{l}$

Still has to be over a closed path, $l$.

$$\sum B_{\parallel} \Delta l = \mu_0 I_{in}$$

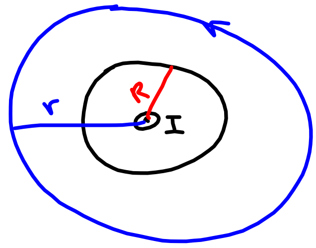
## Applying Ampere's Law to a current carrying wire

I is uniformed distributed across the cross sectional area.




$r > R$

$I_{in}$ - current passing through the path.




$$\Sigma B_{\parallel} \Delta l = \mu_0 I_{in}$$

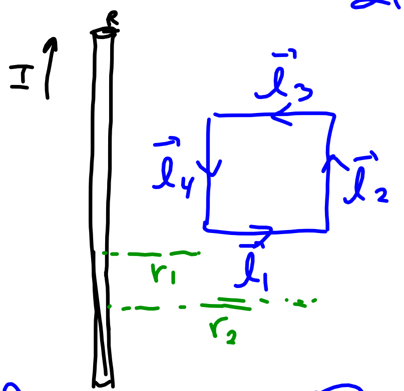
$$B_\phi(r)\, 2\pi r = \mu_0 I$$

$$B_\phi = \frac{\mu_0 I}{2\pi r} \hat{\phi}$$

$$\vec{B} = B_r \hat{r} + B_\phi \hat{\phi} + B_z \hat{z}$$

By symmetry

$$\vec{B} = \vec{B}(r)$$




$l_2 = l_4$

$l_1 = l_3$

$$\Sigma B_{\parallel} \Delta l = \mu_0 I_{in} = 0$$

$$\cancel{\vec{B}_r(r) \cdot \vec{l}_1} + B_z(r_2)\, l_2 - \cancel{B_r(r) \vec{l}_3} - B_z(r_1)\, l_4 = 0$$

$$\underbrace{B_z(r_1) = B_z(r_2)}$$

true for any $r_1, r_2 > R$

so $B_z$ = constant

let $r_2 \longrightarrow \infty$

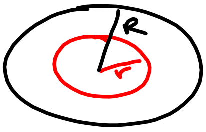
$B_z(\infty)$ must be $0$

$\therefore\ B_z = 0$

We can't use Ampere's Law to show $B_r = 0$ but it does.

Inside the wire

$\vec{I} \odot$



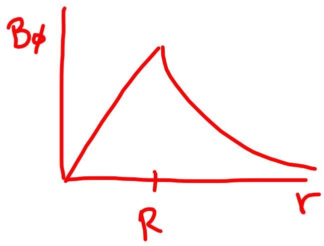

$I_{in} = I \cdot \frac{\cancel{\pi} r^2}{\cancel{\pi} R^2}$

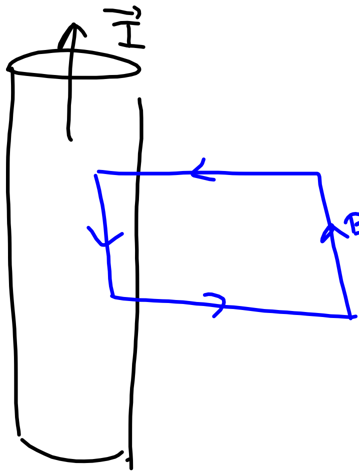
$= I \frac{r^2}{R^2}$

$\sum B_{\parallel} \Delta l = \mu_0 I_{in}$

$B_\phi(r) \cdot 2\pi \cancel{r} = \mu_0 I \frac{r^{\cancel{2}}}{R^2}$

$B_\phi(r) = \frac{\mu_0 I}{2\pi R^2}\, r$

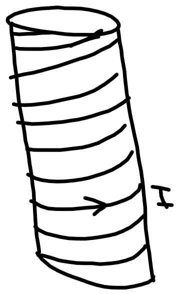


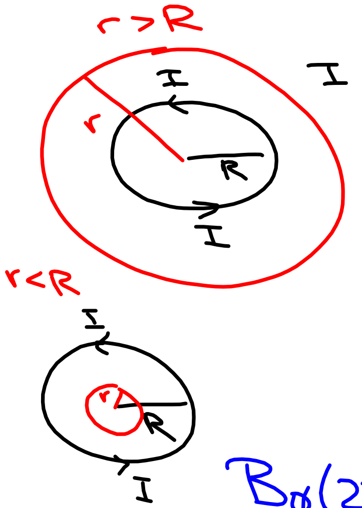




$\sum B_{\parallel} \Delta l = \cancel{\mu_0 I_{in}}$

$B_z(r)\quad r<R =$

Long. Solenoid, radius R, N loops per unit length
current I.




I drifts out of page

$$\sum B_{\parallel} \Delta l = \mu_0 I_{in}$$

$$B_\phi (2\pi r) = \mu_0 I$$

$$B_\phi = \frac{\mu_0 I}{2\pi r}$$

$B_\phi(2\pi r) =$ ☺

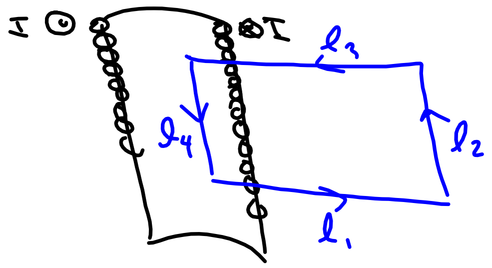
$B_\phi =$ ☺ $r < R$.

Outside

$B_z \rightarrow$ as for a wire $\Rightarrow B_z = 0 \quad r > R$.

$N = \frac{n}{\ell}$

Inside




$$\Sigma B_{\parallel} \Delta \ell = \mu_0 I_{in}$$

$$B_z \cdot \cancel{\ell_4} = \mu_0 \frac{n}{\ell} \cdot \cancel{\ell_4} I$$

$$B_z = \mu_0 N I$$