Cylindrical Coordinate Systems and Ampere's Law

Cartesian Coordinates and Unit Vectors

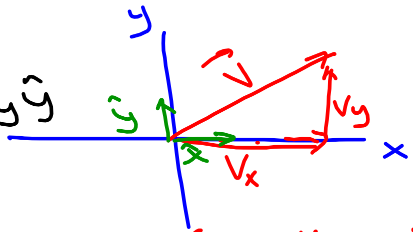
$W = \vec{F}\cdot\vec{d}$

2D

$|\vec{V}|^2 = V_x^2 + V_y^2$

$\vec{V}\cdot\vec{V} = V_x^2 + V_y^2$

$\vec{W} = W_x\hat{x} + W_y\hat{y}$



$\vec{V} = \vec{V}_x + \vec{V}_y$

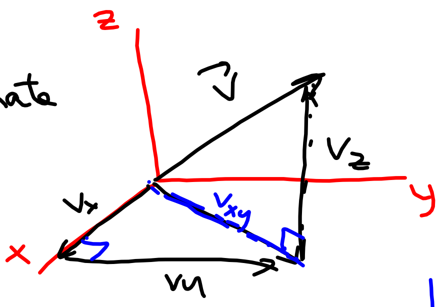
$= V_x\hat{x} + V_y\hat{y}$

$\hat{x}$ = a vector of length 1 in the increasing x-direction

$\vec{V} + \vec{W} = (V_x + W_x)\hat{x} + (V_y + W_y)\hat{y}$

3D

RH coordinate system



$\hat{z} = \hat{x}\times\hat{y}$

$\hat{y}\times\hat{x} = -\hat{z}$ (RH rule)

$\vec{V} = V_x\hat{x} + V_y\hat{y} + V_z\hat{z}$

$|\vec{V}|^2 = V_x^2 + V_y^2 + V_z^2$

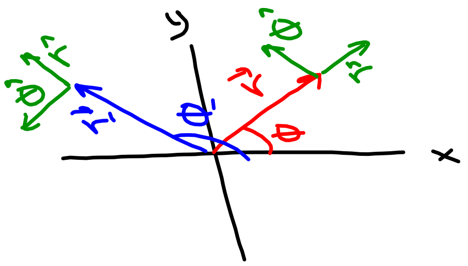
$= \vec{V}\cdot\vec{V}$

$V_{xy}^2 = V_x^2 + V_y^2$

$|\vec{V}|^2 = V_{xy}^2 + V_z^2$

$= V_x^2 + V_y^2 + V_z^2$

2D Polar Coordinate System and Unit Vectors



$\vec{r} = r\,\hat{r}$

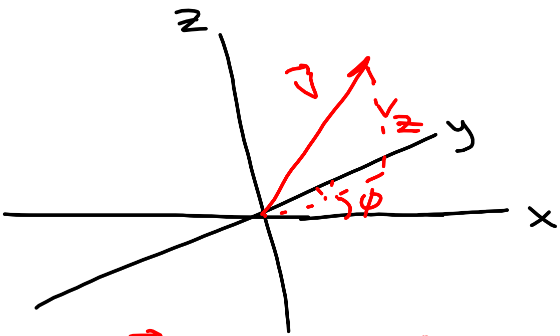
$\hat{r}(\theta)$ = radially outward

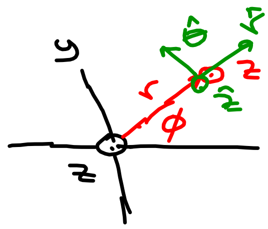
$\hat{\theta}(\theta)$ = CCW

$-\hat{\theta}$ = CW

$-\hat{r}$ = radially inward.

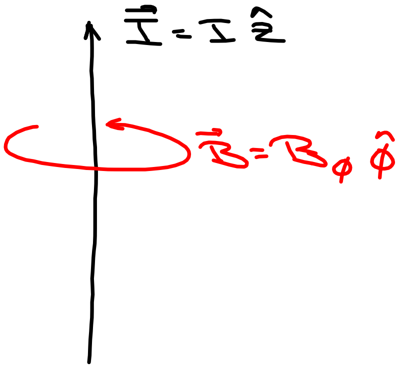
Cylindrical Coordinate Systems and Unit Vectors





$\vec{V} = V_r \hat{r} + V_z \hat{z}$ $\hat{r} = \hat{r}(\phi)$

Applying Cylindrical Coordinate Systems to a Current Carrying Wire



Ampere's Law

Ampere's Law describes the relationship between any current carrying wire and the magnetic field.

sum of an infinite number of things

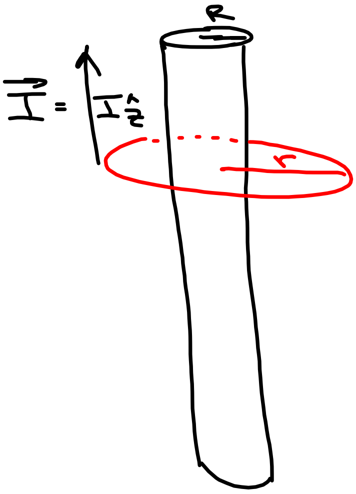
over a closed path

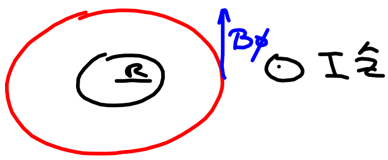
$$\oint \underbrace{\vec{B} \cdot d\vec{l}}_{\text{product of the component of } \vec{B} \parallel \text{ to some small } \Delta l} = \mu_0 \underbrace{I_{in}}_{\text{current enclosed by an arbitrary closed path in space.}}$$

$$\sum B_{\parallel} \Delta l = \mu_0 I_{in}$$

add up all the $B_{\parallel} \Delta l$ s (over a closed path)

Applying Ampere's Law to a current carrying wire (long)





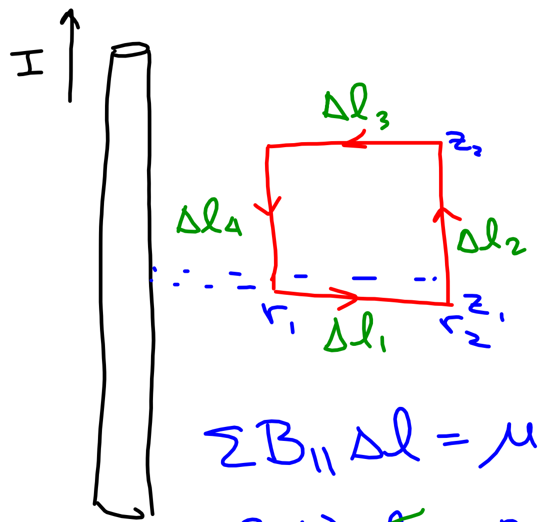
$$\vec{B} = B_r\hat{r} + B_\phi\hat{\phi} + B_z\hat{z}$$

By symmetry $\vec{B} = \vec{B}(r)$

$$\Sigma B_{\parallel}\Delta l = \mu_0 I_{in}$$

$$B_\phi(r)\,2\pi r = \mu_0 I$$ ← all of current is enclosed by the circle.

$$B_\phi(r) = \frac{\mu_0 I}{2\pi r}$$



$\Delta l_1 = \Delta l_3$
$\Delta l_2 = \Delta l_4$

$$\Sigma B_{\parallel} \Delta l = \mu_0 I$$

$$\cancel{B_r(r)\Delta l_1} + B_z(r_2)\Delta l_2 - \cancel{B_r(r)\Delta l_3}$$
$$- B_z(r_1)\Delta l_4 = \mu_0 I_{in} = 0$$

$$B_z(r_1) = B_z(r_2)$$

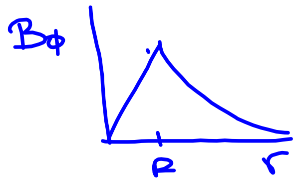
B_z = constant. true for $r>R$.

let $r_2 \to \infty$ $B_z(\infty)$ must $= 0$

$\therefore B_z = 0$ for $r>R$.

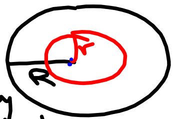
$B_r = 0$ can't be shown with Ampere's law

What about inside the wire?



$r < R$

I ⊙ uniformly distributed in the wire

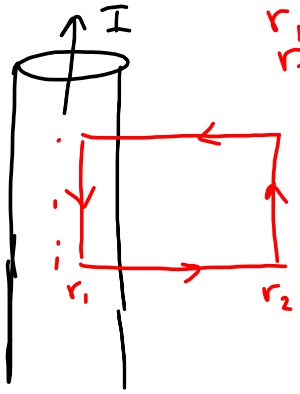


$$\Sigma B_{\parallel} \Delta l = \mu_0 I_{in}$$

$$B_\phi (2\pi r) = \mu_0 I \frac{r^2}{R^2}$$

$$I_{in} = I \frac{\cancel{\pi} r^2}{\cancel{\pi} R^2}$$

$$B_\phi = \frac{\mu_0 r}{2\pi R^2} I$$



$r_1 < R$
$r_2 > R$

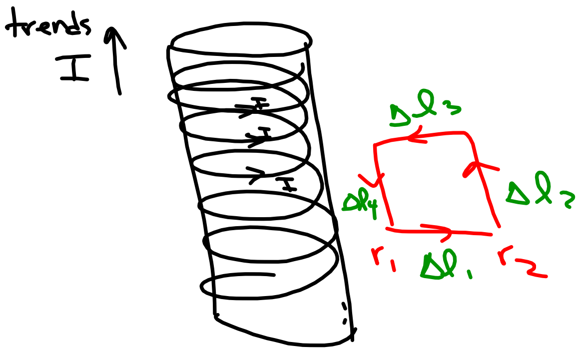
$$\Sigma B_{\parallel} \Delta l = \cancel{\mu_0 I_{in}}$$ ☺

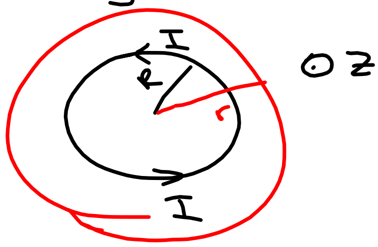
same as before

$B_z = $ ☺

Solenoid, radius R, N loops per unit length

so $N = \frac{n}{l}$ ← # loops ← length





$r > R$

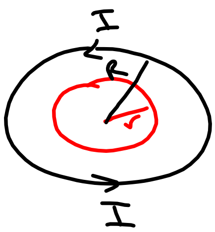
$B_z =$ ☺ like before

$$\sum B_{\parallel} \Delta l = \mu_0 I_{in}$$

$$B_\phi 2\pi r = \mu_0 I$$

$$B_\phi = \frac{\mu_0 I}{2\pi r}$$

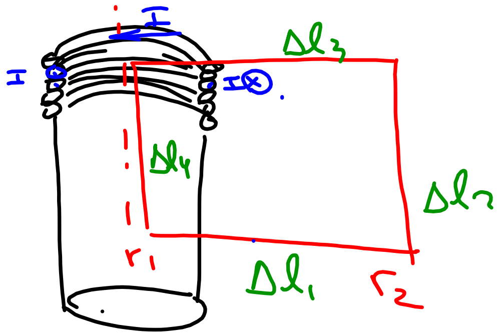
$r < R$



$$\sum B_{\parallel} \Delta l = \cancel{\mu_0 I_{in}}$$

$$B_\phi (2\pi r) = ☺$$

$$B_\phi = ☺$$



$$\sum B_{\parallel} \Delta l = \mu_0 I_{in}$$

$$\cancel{B_{r}(r_1)\Delta l_1} + \cancel{B_z(r_2)\Delta l_2} - \cancel{B_r(r)\Delta l_3} - B_z(r_1)\underbrace{\Delta l_1}_{-l} = \mu_0 I_{in}$$

$$B_z\, l = \mu_0 \frac{n}{\cancel{l}} \cdot \cancel{l}\, I$$

$$\underline{B_z = \mu_0 N I}$$