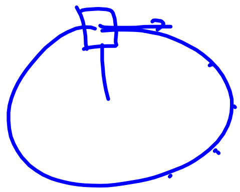
Uniform Circular Motion

Textbook Ch. 11.2

Learning Objectives:
- Students will be able to explain the forces involved in uniform circular motion, demonstrated through free body diagrams and written/verbal explanation.
- Students will analyze, predict and explain circular motion demonstrated through problem solving and experiment.
- Students will demonstrate an understanding of the relationship between period and frequency of an object in circular motion and use this understanding in experiment and problem solving.

Circular Motion



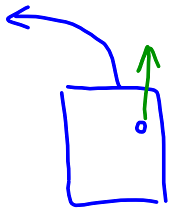
For an object going in a circle

A) what is the direction of the velocity?

B) what is the direction of the force acting on the object?

C) is there an acceleration? And if so, what is the direction of the acceleration of the object?

If you are that object what do you feel? What forces do you experience?

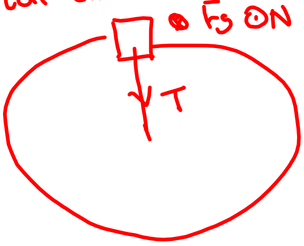


Centripetal vs. Centrifugal Force

center seeking (directed inward) → center fearing (directed outward)

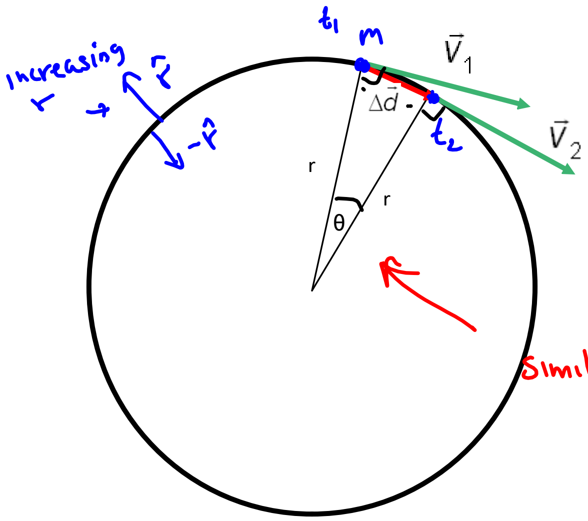
The centripetal force = the net force causing a mass to travel in a circular path.

horizontal circle



$\Sigma\vec{F} = \vec{T} = \vec{F}_c$ ← the centripetal force

Derivation of Centripetal Acceleration Equation



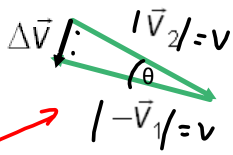
uniform circular motion

$\vec{a} = \frac{d\vec{v}}{dt}$

$|\vec{v}_1| = |\vec{v}_2| = v$

$t_2 - t_1 = t$

$\vec{a}_{ave} = \frac{\vec{V}_f - \vec{V}_i}{t} = \frac{\Delta\vec{V}}{t}$



Similar Δs

$\frac{|\Delta\vec{v}|}{V} = \frac{|\Delta\vec{d}|}{r}$

$\Delta V = \frac{V\Delta d}{r}$

$a_c = \frac{\frac{V\Delta d}{r}}{t}$

$= \frac{V}{r} \cdot \underbrace{\frac{\Delta d}{t}}_{V}$

$F_c = \frac{mv^2}{r} = ma_c \leftarrow \boxed{a_c = \frac{V^2}{r}}$

$\vec{F}_c = -\frac{mv^2}{r}\hat{r}$

$\vec{a}_c = -\frac{V^2}{r}\hat{r}$

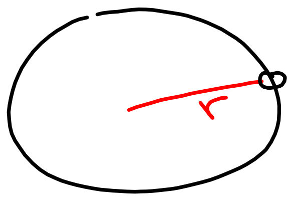
radially inward.

Uniform circular motion

$a_c = \frac{v^2}{r}$

$F_c = \frac{mv^2}{r}$

apply at any moment in time



The time to complete one revolution is called the period, T.

$v = \frac{2\pi r}{T}$

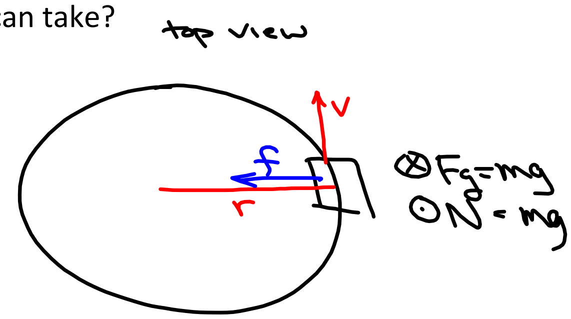
$$a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{\frac{4\pi^2 r^{\cancel{2}}}{T^2}}{\cancel{r}}$$

$$\boxed{a_c = \frac{4\pi^2 r}{T^2}} \quad \text{so} \quad \boxed{F_c = \frac{m\,4\pi^2 r}{T^2}}$$

uniform circular motion only

Example

A car (m=1000 kg) takes a turn at 40 m/s. If the coefficient of friction between the road and the tires is 0.90, what is the minimum radius the car can take?



$\mu = 0.90$

$$\Sigma F : F_c = f = \frac{mv^2}{r}$$

$$\mu N = \frac{mv^2}{r}$$

$$\mu \cancel{m} g = \frac{\cancel{m} v^2}{r}$$

$$r = \frac{(40\,\text{m/s})^2}{0.9(9.8\,\text{m/s}^2)}$$

$$= 181\,\text{m}$$

Homework: Sheet #1-5