

Centripetal Motion and Universal Gravitation

Circular Motion

Centripetal
Center Seeking

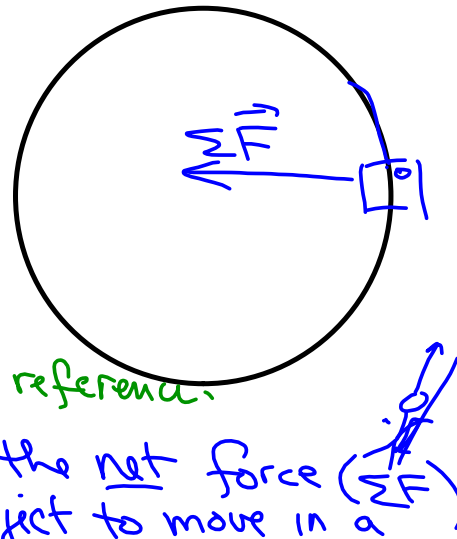
vs.

~~Centrifugal~~
~~Center Fearing~~

Artificial force trying to
apply Newton's laws in
an accelerated frame of reference.

Centripetal force: the net force ($\Sigma \vec{F}$)
that causes an object to move in a
circle

$$\Sigma \vec{F} = \vec{F}_c = -F_c \hat{r}$$



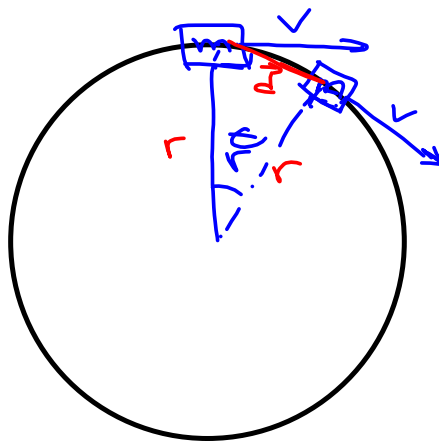
Centripetal Acceleration

assume $v = \text{const.}$

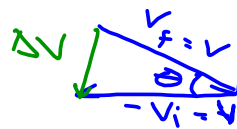
$$a_c = \frac{v}{r} = \frac{2\pi r}{T} \cdot \frac{1}{r} = \frac{2\pi}{T}$$

$$a_c = \frac{v^2}{r}$$

$$F_c = \frac{mv^2}{r}$$



$$a = \frac{\Delta v}{t}$$



$$\frac{v}{r} = \frac{\Delta v}{v t} \quad , \text{ but } \vec{a} = \frac{\vec{\Delta v}}{t}$$

$$\frac{v}{r} = \frac{\Delta v}{v t} \Rightarrow \frac{v^2}{r} = \frac{\Delta v}{t} = a$$

$$\vec{a}_c = -\frac{v^2}{r} \hat{r}$$

$$\vec{F}_c = -\frac{mv^2}{r} \hat{r}$$

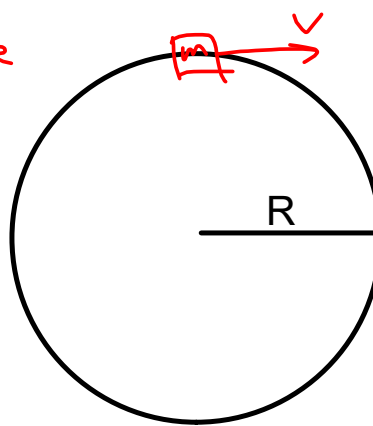
If it takes a time, T ,
to go around the circle

$$v = \frac{2\pi R}{T} = \frac{d}{T}$$

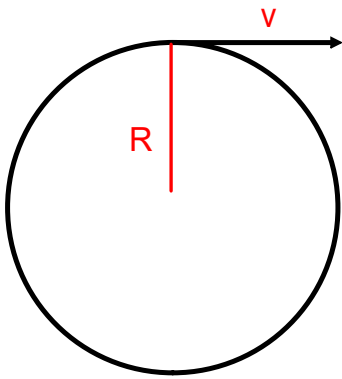
$$a_c = \frac{v^2}{R} = \frac{\left(\frac{2\pi R}{T}\right)^2}{R}$$

$$a_c = \frac{4\pi^2 R}{T^2}$$

$$\text{So } F_c = \frac{m 4\pi^2 R}{T^2}$$



Centripetal Force



Sample Question: Car going around a turn

The diagram shows a car of mass m on a turn of radius R . A dashed line represents the radius R from the center of the turn to the car. A red arrow labeled f points towards the center, representing friction. A curved arrow labeled v indicates the car's velocity. To the right of the car, the following equations are written:

$$\Sigma F = f = F_c$$

$$f_{max} = \mu mg = \frac{mv_{max}^2}{R}$$

$$F_g = mg$$

$$N = F_g$$

$$v_{max} = \sqrt{\mu g R}$$

Below the diagram, the following calculations are shown:

$$\text{So } v = 110 \frac{\text{km}}{\text{h}} = \frac{110000 \text{ m}}{3600 \text{ s}} = 30.6 \frac{\text{m}}{\text{s}}$$

$$R = \frac{v_{max}^2}{\mu g} = \frac{35^2}{0.5 \cdot 10} = \underline{\underline{250 \text{ m}}}$$

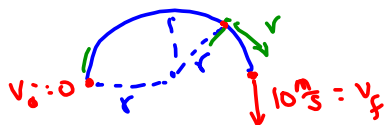
Tangential Acceleration:
Accelerating around a circle

$$v_o = v_i = v_f$$

$$v_o = v(\odot) = v \text{ at time } \odot$$

Car accelerates from rest to 10 m/s while completing a semicircle of radius 40 m.

What is the net acceleration $\frac{3}{4}$ of the way around the semicircle?



$$a_T = \frac{v_f^2 - v_i^2}{2d} = \frac{100}{2(\pi(40))} = \frac{5}{4\pi} \text{ m/s}^2$$

