Centripetal Motion and Universal Gravitation

Circular Motion

centripetal

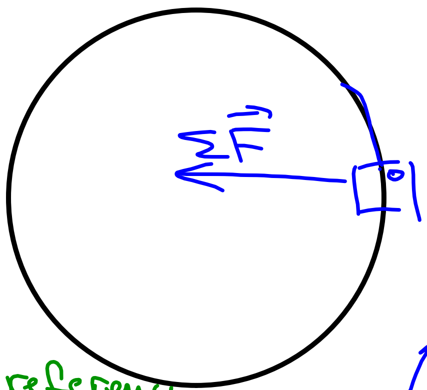
center seeking

vs.

~~centrifugal~~

~~center fearing~~

Artificial force trying to apply Newton's laws in an accelerated frame of reference.



Centripetal force: the net force ($\Sigma \vec{F}$) that causes an object to move in a circle

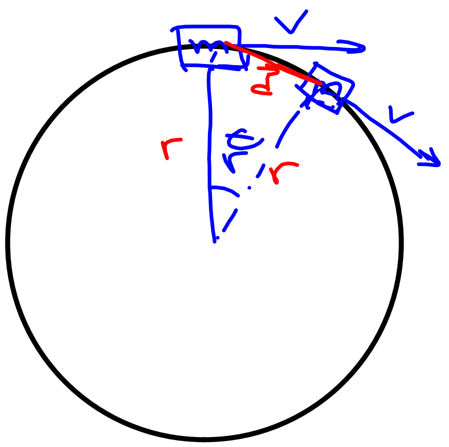
$$\Sigma \vec{F} = \vec{F}_c = -F_c \hat{r}$$

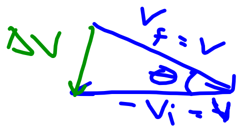
Centripetal Acceleration

assume $v = \text{const.}$

$$\vec{a}_c = \frac{\Sigma \vec{F}}{m} = \frac{\vec{F}_c}{m}$$

$$\vec{a} = \frac{\Delta \vec{v}}{t}$$





$$\frac{d}{r} = \frac{\Delta v}{v}$$, but $\vec{d} = \vec{v}t$

$$\frac{vt}{r} = \frac{\Delta v}{v}$$

$$\frac{v^2}{r} = \frac{\Delta v}{t} = a$$

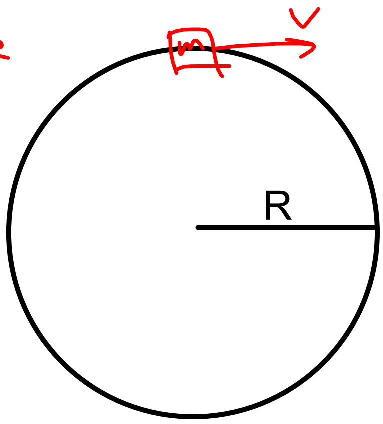
$$a_c = \frac{v^2}{r}$$

$$F_c = m\frac{v^2}{r}$$

$$\vec{a}_c = -\frac{v^2}{r}\hat{r}$$

$$\Sigma\vec{F} = \vec{F}_c = -\frac{mv^2}{r}\hat{r}$$

If it takes a time, T, to go around the circle



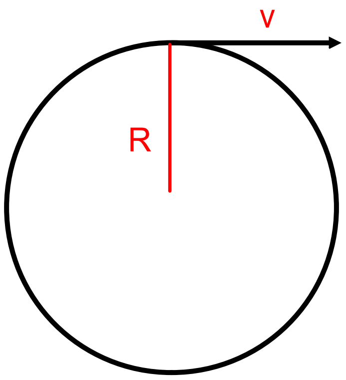
$$V = \frac{2\pi R}{T} = \frac{d}{T}$$

$$a_c = \frac{V^2}{R} = \frac{\left(\frac{2\pi R}{T}\right)^2}{R}$$

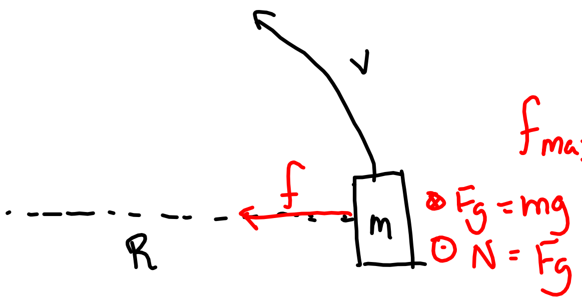
$$a_c = \frac{4\pi^2 R}{T^2}$$

So $F_c = \frac{m\, 4\pi^2 R}{T^2}$

Centripetal Force



Sample Question: Car going around a turn



$$\Sigma F = f = F_c$$

$$f_{max} = \mu mg = \frac{m v_{max}^2}{R}$$

$$v_{max} = \sqrt{\mu g R}$$

So $v = 110 \frac{km}{h} = \frac{110\,000\ m}{3600\ s} = 30.6 \frac{m}{s}$

$$R = \frac{v_{max}^2}{\mu g} = \frac{35^2}{0.5 \cdot 10}$$

$$\approx \underline{\underline{250 m}}$$

Tangential Acceleration:
Accelerating around a circle

$v_0 = ☺ = v_i$ $\qquad$ $v_0 = v(☺) = v$ at time ☺

Car accelerates from rest to $10\,\text{m/s}$ while completing a semicircle of radius 40m.

What is the net acceleration $\frac{3}{4}$ of the way around the semicircle?

$$a_T = \frac{v_f^2 - v_i^2}{2d} = \frac{100}{2(\pi(40))} = \frac{5}{4\pi}\ \text{m/s}^2$$