Light: Geometric Optics

Regular and Diffuse Reflection

Sections 23-1 to 23-2.

How We See

We see because light reaches our eyes. There are two ways, therefore, in which we see: (1) light from a luminous object reaches our eyes directly (such as from a light bulb); or (2) the light is reflected off an object and reaches our eyes.

In the absence of light we have darkness. Darkness appears... well, black. Black is the "colour"we see when no light is reflected to our eyes. White, on the other hand is the colour we see when all (or many) colours of light are reflected to our eyes. We see colours based upon the colour of light reflected to our eyes.

There are two types of reflection. The type of reflection we normally see is diffuse reflection. Light is reflected off "bumpy" surfaces in all directions. This is also known as scattering. The second type of reflection, which we see off a mirror, for example, is called regular reflection, so called because it is predictable. In both cases, the light obeys the law of reflection, but in one case the surface is "bumpy" and in the other, the surface is "smooth". Diffuse reflection, while most common, is too unpredictable. We prefer, rather, to focus on regular reflection.

There is considerable evidence that indicates that light travels in straight lines (or almost straight lines) in a variety of circumstances (shadows for instance). We have depth perception, for example, because we are able to triangulate a location with two eyes (try this... with only one eye open you will have difficulty grabbing onto small objects, or objects with which you are not familiar). This is based on the assumption that light travels in straight lines. Surveying engineers do precisely the same thing on a larger scale to locate objects, measure distances, etc. Light travelling in straight lines has led us to the ray model of light, that is light travels in straight paths called rays.

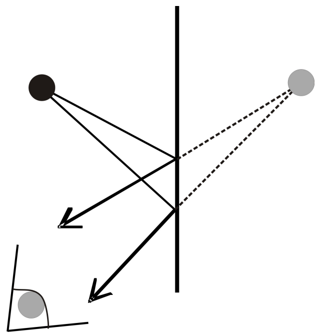


Figure 1.1: Virtual Image

Regular Reflection and Plane Mirrors

SINCE AN ORDINARY OBJECT SCATTERS LIGHT IN ALL DIRECTIONS, we can consider one, two or more rays of light coming from the object. Consider an object, represented by the black dot, scattering light in front of a mirror (see Figure 1.1). If we consider only two rays scattered by the object, when they hit the mirror, they will obey the law of reflection. Consider, then, an eye looking at the reflected light. It appears to the eye as if the light has actually come from behind the mirror (as indicated by the dotted lines). In fact the eye is tricked into believing the object is behind the mirror. What the eye is actually seeing is an image of the object. This type of image is called a virtual image because there is nothing really there.

The image appears at the intersection of the two lines of sight. If we call the distance from the object to the mirror the object distance, d_o, and the distance from the image (represented by the grey circle) to the mirror the image distance, d_i, it can be shown for a plane (or flat mirror) that $d_o = d_i$.

Question for thought: How long must a mirror be for a person of height h to see himself fully in the mirror?

Spherical Mirrors

Sections 23-3

Spherical Mirrors

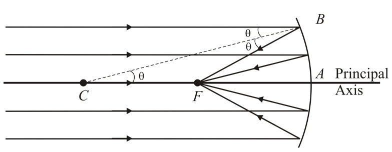


Figure 1.2: Concave mirror with parallel incoming rays reflected through the focal point

SPHERICAL MIRRORS COME, as their title indicates, from a sphere. If we were to take a shiny ball and slice part of it off, we would have the potential for two types of mirrors: the shiny outside part of the ball and the shiny inside part of the ball. The mirror shiny on the outside of the sphere is called a convex mirror, whereas the one shiny on the inside is called a concave mirror.

Concave Mirrors

CONSIDER AN IMAGINARY LINE connecting the center of the mirror's surface to the center of the sphere, called the ***center of curvature***, C (i.e., a radius). This line extended in both directions is called the principal axis (see Figure 1.2). If we consider rays of light travelling toward the mirror parallel to the principal axis, they will reflect to focus at a point.2in^0 , obeying the law of reflection. This point is called the principal focus, or the focal point, F. The distance from the mirror to the focal point is called the focal length, f.

[0] Note: This is true only as long as the width of the mirror is small compared to the radius of curvature, r.

From Figure 1.2, we can see that CFB is an isosceles triangle, so $CF = FB$. We know $CB = CA$ since both are radii. Since width of the mirror is small compared to CA, we realize that FB is almost equal to FA, so $CF \approx FA$ ($\approx$ means "approximately equal to"), which is the ***focal length***, f. So $2f = r$, or

$$f = \frac{r}{2}.$$

We now consider an object some arbitrary distance, d_o, in front of the mirror. We recall that the object reflects light in all directions, and we have a variety of rays to choose from. Obviously to diagram many would be virtually impossible, so we will choose a representative few. In fact, there are 3 rays we can select easily without having to pull out a protractor to measure angles, as shown in Figure 1.3. The image is formed where the rays of light meet. Any two are sufficient to locate the image.

Note that the image formed here is different than the one formed by the plane mirror. In the case the rays of light actually meet. This is called a real image and has the ability to be projected onto a screen. Note also that we can locate the image using any two of these three rays. There are six different cases for concave mirrors, related to different positions of the image: $d_0 > r, d_0 = r, r > d_0 > f, d_0 = f$ and $d_0 < f$. We will examine these in class.

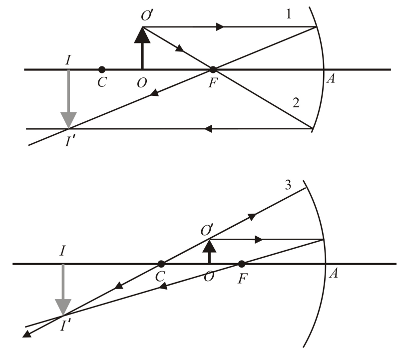


Figure 1.3: The first ray (1) enters parallel to the principle axis and is reflected through the F. The second ray (2) enters through F and reflects parallel to the principle axis. The third ray (3) passes through C. Since this is a radius, it is perpendicular to the mirror, so $\theta_i = \theta_r = 0$

The Mirror Equation

Geometrically, we can derive an equation linking the object distance, d_0, the image distance, d_i, and the focal length, f. This is called the mirror equation. In Figure 1.4, we can see that $O'AO \sim$ (“is similar to") $I'AI$. Therefore, we have the relationship

$$\frac{I'I}{O'O} = \frac{IA}{OA}$$

or defining the object height and image height to be h_o and h_i respectively,

$$\frac{h_i}{h_o} = \frac{d_i}{d_o}.$$

We also can see that $O'OF \sim$ (approximately)BAF. This gives us the relationship

$$\frac{BA}{O'O} = \frac{AF}{OF} \qquad \text{or} \qquad \frac{h_i}{h_o} = \frac{f}{d_o - f}.$$

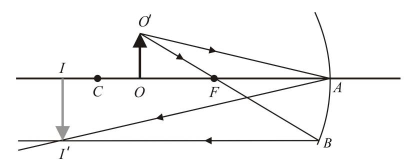


Figure 1.4: Using similar triangles to derive the mirror equation.

By combining these two equations we get

$$\frac{d_i}{d_o} = \frac{f}{d_o - f}.$$

By multiplying these out and dividing all terms by $d_o d_i f$, we get

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}. \tag{1.1}$$

Magnification

The magnification of a mirror is defined simply by the ratio of the image height as compared to the object height, i.e.,

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}, \tag{1.2}$$

where the negative sign is to take into account the conventions in Table 1.1.

Table 1.1: Conventions for Mirrors

Quantity	+	−
$\mathbf{d_o, d_i}$	in front of mirror	behind mirror
f	in front of mirror	behind mirror
$\mathbf{h_o, h_i}$	erect	inverted
m	erect	inverted

Convex Mirrors

CONVEX MIRRORS FOLLOW all the same "rules" as concave mirrors, except the center of curvature (and therefore the focal point) is on the back side of the mirror. Obviously rays of light do not reflect through the focal point, but they reflect as though they have come from the focal point, as shown below. Unlike concave mirrors, there is only one case for convex mirrors. The image will always be virtual, erect and smaller.

Lenses

Section 23-7 to 23-9.

WHEN LOOKING AT LENSES, we are considering only spherical lenses. These lenses have surfaces which are, by definition spherical in nature. Like spherical mirrors, they fall into two categories: convex and concave. While there are different types of convex and concave lenses (double, plano, and concavo or convexo) we will consider primarily double lenses (recognizing that the others have similar features).

Lenses behave much as mirrors do, except they work on the principal of refraction rather than reflection. The mathematics are the same (the mirror equation is also the lens equation), with slightly different conventions, as shown in Table 1.2.

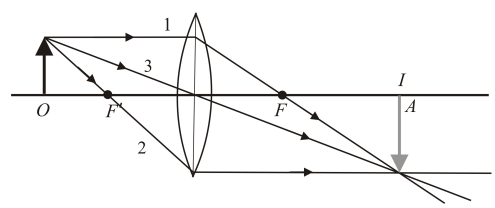
Table 1.2: **Conventions for Lenses**

Quantity	+	−
$\mathbf{d_o}, \mathbf{d_i}$	real	virtual
$\mathbf{f}$	convex	concave
$\mathbf{h_o}, \mathbf{h_i}$	erect	inverted
$\mathbf{m}$	erect	inverted

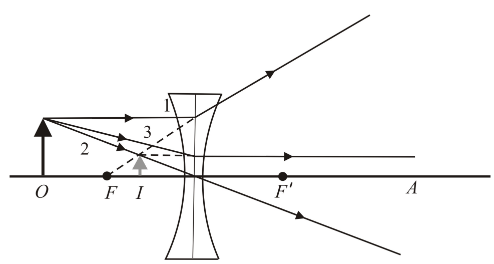
As can be seen by the conventions, a concave mirror behaves in a similar fashion to convex lenses, and vice versa. Therefore, much as there are six cases for concave mirrors, there are six cases for convex lenses. Likewise, there is only one case for concave lenses.

Ray Tracing

Figure 1.5: Ray tracings of convex and concave lenses



(a) Convex lens



(b) Concave lens

Figures I.5(a) and I.5(b) depict ray tracings of convex and concave lenses respectively. In each case, there are three rays that we know how they behave. Any two is sufficient to locate an image. For convex lenses, a ray travelling in parallel to the principal axis will refract through the far focal point (1); a ray through the near focal point will refract parallel to the principal axis (2); and a ray through the optical centre passes straight through the lens (3). The rays for concave lenses are similar: (1) a ray coming in parallel to the principal axis refracts from the near focal point; (2) a ray travelling toward the far focal point refracts parallel to the principal axis; and (3) a ray will travel straight through the optical centre.

Combination Lenses

Section 23-10.

Multiple Lenses

TO DEAL WITH MULTIPLE LENSES, we deal with one lens at a time. Consider two convex lenses some distance apart, with an object in front of one lens (the *objective* lens). The first lens forms an image (either real or virtual). The second lens then acts upon the rays of light that pass through the first lens. This is the equivalent of treating the image of the first lens as the object of the second lens.

We treat the ray tracings in the same way. We construct the diagram for the first lens. We then effectively *ignore* the first lens and draw the ray tracing for the second lens. While this may cause some confusion, if we draw the diagram, we see that those rays of light *actually exist*. We will consider a couple of examples in class, such as a microscope.

THE ONLY SERIOUS DIFFICULTY with multiple lenses comes from the case of a virtual object. This occurs when the image from the first lens doesn't get a chance to form before the rays of light hit the second lens. We will investigate this in class as a third example. The complication is related to the ray diagram – we don't know how the rays of light *behave*. Mathematically, however, we find the distance the image *would have* formed, had the second lens not been there, and find the distance from that would-be image to the second lens, making d_o negative for that lens. The final image location is then determined using the lens equation (with the focal length f_2 and the negative value of d_{o_2}).