Electric Potential, Energy and Capacitance

Electric Potential, Potential Difference and the Electron Volt

Sections 17-1 to 17-4

Electric Potential and Potential Difference

Let us consider the change in gravitational potential energy as a mass is moved from a height h to the ground. We know that the work done by gravity in moving the mass, $W = -\Delta PE$. Further, we know that gravitational energy is given by $PE = mgh$ near the surface of the earth. We could define a gravitational potential which is independent of the mass, i.e. equal to PE/m. That is, both a mass of 100 kg and a mass of 100 g would have the same gravitational potential at the same height, since the potential difference would be dependent only on the acceleration of gravity and the height.

We do the same thing with electricity. We define an electric potential, V such that it is independent of the charge. At some point, a, a charge, q, has an electric potential of

$$V_a = \frac{PE_a}{q}. \tag{7.1}$$

By moving a charge from one potential to another (say to V_b), we can determine the potential difference (ΔV or V_{ba}) by subtracting the potentials (much like with gravity), i.e.,

$$V_{ba} = V_b - V_a = \frac{\Delta PE}{q}.$$

If we define W_{ba} as the work to move a charge from point a to point b, then the potential difference is also given by

$$V_{ba} = \frac{W_{ba}}{q}. \tag{7.2}$$

The unit of electric potential and potential difference is the ***volt***, V, name after Alessandro Volta, who is credited with constructing the first voltaic pile. One volt is equal to a Joule per Coulomb ($1\ \mathrm{V} = 1\ \frac{\mathrm{J}}{\mathrm{C}}$).

Electric Potential and Electric Field

Now if we consider this electric force to be supplied by a constant electric field, E, such as between two parallel plates. We know that the work done in moving a charge from point a to b from equation 7.2, but work is also given by $W = Fd$ and the electric force is given by $F = qE$ from equation 6.4. Combining these we get

$$W = Fd = qEd.$$

where d is the distance distance parallel to the electric field that the charge has been moved. By equating these two equations for work we get

$$V_{ba} = Ed. \tag{7.3}$$

If we then consider two parallel plates with a separation d connected to a battery of potential difference, V, we can find the electric field between these two plates with the above equation.

Equipotential Lines

Equipotential lines are analogous to contour lines on a topographical map. Much like contour lines show areas at the same height (i.e. the same gravitational potential), equipotential lines show spaces at the same electric potential. When comparing with electric field lines, equipotential lines are always at right angles to the field lines (since the field lines show the direction of the force, in order for no work to be done, there must be no force). For examples see diagrams on p. 507 of Giancoli.

The Electron Volt

We know the SI unit for energy is the Joule. When dealing with small charges however, the Joule is an excessively large unit. Instead, we use the unit of the electron volt. Since $W = qV$, one electron volt (eV) is defined as the energy acquired by one elementary charge moving through an electric potential of 1 V, i.e., $W = (1\text{ e})(1\text{ V}) = 1\text{ eV}$.

In terms of Joules, $1\text{ eV} = (1.602x10^{-19}\text{ C})(1\text{ V}) = 1.602x10^{-19}$ J.

Electric Potential Due to a Point Charge

Section 17-5

We know that $V = \frac{\Delta PE}{q}$ defines the electric potential, and that $W = \Delta PE$. We also know that $W = F_{ave}d$ in general. We can come to an understanding of the electric potential due to a point charge, therefore, by viewing the graph of F vs. r, where r is the radial distance from the central charge. It is an inverse square graph (since $F = \frac{kQ_1Q_2}{r^2}$) and therefore the area under the graph can be determined using calculus. It turns out that

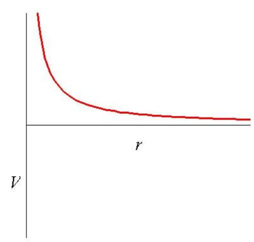
$$PE(r) = \frac{kQ_1Q_2}{r} \tag{7.4}$$

and therefore

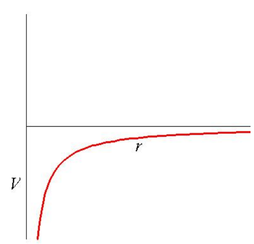
$$V(r) = \frac{kQ}{r}, \tag{7.5}$$

where Q is the central charge.

If we plot $V(r)$ vs. r we get the graphs shown in figure 7.1. Note that the 1st graph has Q positive and the 2nd graph has a negative Q. These are often called potential wells.



(a) $V = \frac{kQ}{r}$, $Q > 0$



(b) $V = \frac{kQ}{r}$, $Q < 0$

Figure 7.1: Plots of the electric potential versus distance for positive and negative charges

You should notice that in either case, $V(\infty) = 0$. We choose $r = \infty$ as reference point for because the potential, the potential energy and the electric field and force are all equal to zero.

If we were to look at the work required to move a charge q from a distance a to a distance b from a charge Q, we would see that

$$W = qV_{ba} = q\left[V\left(b\right) - V\left(a\right)\right] = q\left[\frac{kQ}{b} - \frac{kQ}{a}\right]$$

or

$$W = kQq\left[\frac{1}{b} - \frac{1}{a}\right]$$

Potential from Multiple Charges

It should be noted that potential is a scalar quantity, so the total potential from multiple charges is the algebraic sum (as opposed to the vector sum) of the individual potentials, i.e.

$$V_{total} = \sum_i V_i \tag{7.6}$$

We will investigate a couple of examples in class.

Capacitance, Dielectrics and Energy

Section 17-7 to 17-9

A capacitor (sometimes called a condenser) is simply a device that stores charge. It consists of two conductors (often plates) that are separated by a small distance. They are used in circuits to store charge (and energy), provide backup energy supplies and to help prevent power surges. The capacitance of a capacitor is simply the amount of charge it can store per unit of potential difference, i.e.,

$$C = \frac{Q}{V}, \tag{7.7}$$

or if you prefer, an object's capacity for charge per volt. As the energy per Coulomb (i.e. the potential difference) increases, so does the amount of charge an object can store.

For a parallel plate capacitor, with plates each of area A and separated by a distance d, the capacitance is equal to

$$C = \epsilon_0 \frac{A}{d}$$

if the space in between is a vacuum (where ϵ_0 is the permittivity of free space). Most capacitors, however, have an insulating sheet placed between the plates. This allows a greater capacitance because the charge is less likely to jump across the insulator. This insulating sheet is called a dielectric. It increases the capacitance by a factor K, which is dependent upon the material. For a table of dielectric constants see Table 17-3 on p. 514. In general then,

$$C = K\epsilon_0 \frac{A}{d}$$

or

$$C = \epsilon \frac{A}{d} \tag{7.8}$$

where $\epsilon = K\epsilon_0$ is the permittivity of the material.

Capacitors, since they have the ability to store charge with a potential difference, also have the ability to store energy. Since $W = qV$, the amount of work required to move a charge Q from one plate to the other would be $W = QV$ if the voltage were constant. However, the voltage is proportional to the amount of charge built up on the plates, so it starts at zero and build up to a maximum V. We must therefore look at the average voltage as we gradually put the charge Q on the plates. Obviously, the average voltage is $\frac{1}{2}V$, so we get

$$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} \tag{7.9}$$

Note: This is the first time we have used U as the symbol for energy. It is, in fact, the "standard" symbol for potential energy. The argument for the factor of one half is similar to that for springs.