Electric Currents and DC Circuits

The Basic Circuit and Components

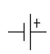
Sections 18-1 to 18-4

Battery

The battery is the object within a circuit that provides electrical energy. It does this through the transformation of chemical energy. The simplest battery (or cell) is made of two different metals (called electrodes) and an electrolyte, such as an acid, salt or basic solution. It should be noted that one of the electrodes could also be carbon.

The chemical reaction within a battery is quite complicated, although it is, at least in part, a redox reaction. As a result a potential difference (V) is established between the two electrodes - the negative terminal, the cathode, and the positive terminal, the anode.

The schematic symbol for a battery is :



Current

When a continuous conducting path is connecting the terminals of a battery we have a circuit. The conductors allow electrons to flow from the negative terminal of the battery to the positive one. This flow of charges results in an electric current (I).

Current is defined mathematically as the rate of flow of charge through a point in the circuit, i.e.,

$$I = \frac{\Delta q}{\Delta t}, \tag{7.1}$$

or more precisely $I = \frac{dq}{dt}$ from a calculus point of view. The unit of current is the ampere (A) after Andre Ampere. Thus $1\ \text{A} = 1\ \frac{\text{C}}{\text{s}}$.

The direction of current is slightly more complicated. Originally (before we knew any better) we defined current as flowing from positive to negative. However, in general, it is NOT the positive charges doing the flowing! When we discovered that it is in fact the electrons flowing, we changed our minds - current flowed with the electrons, i.e. electron flow. This is logical, ***however***, current travels faster than the electrons! Therefore current cannot be the flow of electrons, even though it is caused by the flow of electrons. We therefore reverted to the original definition of current, i.e., ***conventional current***, flowing from positive to negative. We will illustrate this with an analogy in class.

Resistance

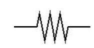
Resistance is the opposition to current flow. Mathematically we define resistance to be the ratio of voltage to current, i.e., $R \equiv \frac{V}{I}$. Georg Simon Ohm discovered that this ratio was in fact constant for metal conductors at a constant temperature. In fact most materials have what we call an "Ohmic" region - a range of Voltages over which the resistance is constant. Rearranging this gives us

$$V = IR, \tag{7.2}$$

known as Ohm's law.

The unit of resistance is the ohm (Ω), where $1\ \Omega = 1\ \frac{\mathrm{V}}{\mathrm{A}} = 1\frac{\mathrm{Js}}{\mathrm{C}^2}$.

While there are different schematic symbols for different types of resistances (a light bulb for example), in a circuit, we can represent a resistance with the resistor symbol:



A connector with negligible resistance (or no resistance in the case of a "superconductor") is indicated with a straight line. So a simple circuit looks like this:

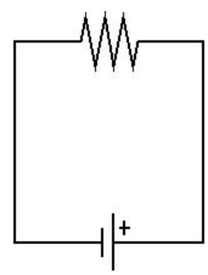


Figure 7.1: A simple circuit containing a battery (source) and a resistor (load) connected by a conductor.

Factors affecting Resistance

There are four factors that affect resistance:

1. Substance - different substances have different resistivities (ρ). See table 18-1, p. 535 for resistivities of substances. Resistance is proportional to the resistivity (i.e., the larger the resistivity of a substance, the larger the resistance).
2. Length - resistance is directly proportional to length (L).
3. Cross sectional area (A) - resistance is inversely proportional to the area. This seems a little counterintuitive, but the option to the current traveling through the conductive substance is having it travel through air (much higher resistivity).
4. Temperature - resistance increases with temperature, but not necessarily linearly. This factor is usually incorporated within the resistivity (see table 18-1 for Temperature coefficients).

As a result, we can say that

$$R = \frac{L}{A}\rho\left(T\right) \tag{7.3}$$

where $\rho\left(T\right)$ represents the resistivity as a function of temperature and

$$\rho\left(T\right) = \rho_0\left(1 + \alpha T\right) \tag{7.4}$$

where ρ_0 is the resistivity at 0°C and α is the temperature coefficient.

Electrical Power, Series & Parallel Circuits

18-6 to 18-7, 19-1

Electrical Power

You will recall that mechanical power was calculated by $P = \dfrac{\Delta E}{\Delta t}$ and that electrical work is calculated as the product of the charge moved through a potential difference, $W = qV$ from equation 7.2, and thus

$$\Delta W = \Delta\left(qV\right) = \Delta qV$$

where V is constant. Combining the two equations, we get

$$P = \frac{\Delta qV}{\Delta t} = V\frac{\Delta q}{\Delta t}$$

Using our definition of current in equation 7.1 we get

$$P = VI. \tag{7.5}$$

Combining this power equation with Ohm's law(7.2), we can get two other forms of the power equation:

$$P = I^2R \tag{7.6}$$

and

$$P = \frac{V^2}{R}. \tag{7.7}$$

Series Circuits

A series circuit is a circuit that contains multiple loads (or resistances) but only one path through which the current can flow. An example of a series circuit is shown in the figure 7.2.

Series circuits have the following characteristics:

1. Ohm's law applies to each element of the circuit as well as the totals.
2. The current is constant throughout the circuit.
3. The total resistance encountered by the current is equal to the sum of the individual resistances.
4. The total voltage (potential difference) is equal to the sum of the individual voltages (p.d.'s).

We will solve an example in class.

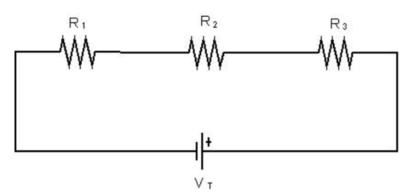


Figure 7.2: Series circuit with three resistors.

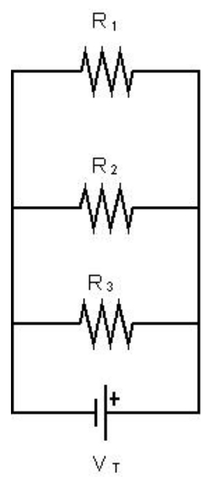


Figure 7.3: Parallel circuit with three resistors.

Parallel Circuits

A parallel circuit is a circuit that contains multiple loads, but only one load in each path through which the current can flow. An example of a parallel circuit is shown in figure 7.3.

Parallel circuits have the following characteristics:

1. Ohm's law applies to each element of the circuit as well as the totals.
2. The voltage is constant across each branch of the circuit.
3. The total current is equal to the sum of the individual currents.
4. The reciprocal of the total resistance is equal to the sum of the reciprocals of the individual resistances.

We will solve an example in class.

EMF, Combination Circuits and Kirchoff's Laws

Sections 19-2 to 19-5

Electromotive Force

Thus far, we have discussed the potential difference of a battery. This is commonly referred to as the ***terminal voltage***, or the voltage measured at the terminals of the battery. This value, however, is dependent upon the resistance attached to the battery. This is because batteries, generators, etc. all have an ***internal resistance***, r. As a result, there is a potential difference *within* the source itself.

The potential difference between the terminals of a source when no current flows to an external circuit is called the EMF or ***electromotive force***. This term is a bit of a misnomer as it is a voltage and not a force.

A real battery is modeled having a perfect source or EMF (ε) in series with an internal resistance, r. The terminal voltage V_T is equal to

$$V_T = \varepsilon - Ir = \varepsilon - V_r. \tag{7.8}$$

We will examine an example in class.

Circuit Reduction

In reality, most circuits are combinations of series and parallel circuits, and are not solved as simply. One means of solving circuits is by a process known as circuit reduction. We combine elements of the circuit until we have a series or parallel circuit and then work backwards. The rules for series circuits apply to elements in series and the rules for parallel circuits apply to elements in parallel.

We will look at two examples in class - one a reduction to a series circuit and one to a parallel.

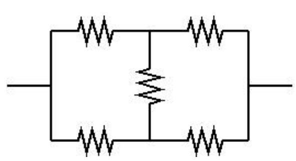


Figure 7.4: Wheatstone bridge

Kirchoff's Laws

Kirchoff's Laws are the most powerful means of solving circuits. While not all circuits can be reduced (a Wheatstone bridge for example: see figure 7.4), Kirchoff's Laws will solve them, by setting up a system of equations and unknowns.

- Kirchoff's Current Law states that the sum of the currents entering any point must equal the sum of the currents leaving the point, i.e.,

$$\sum I_{in} = \sum I_{out}.$$

This is essentially a statement of the law of conservation of charge.

- Kirchoff's Voltage Law states that the sum of the changes in potential difference around any closed path must be equal to zero, i.e.,

$$\sum_{loop} V = 0.$$

This is a statement of the law of conservation of energy.

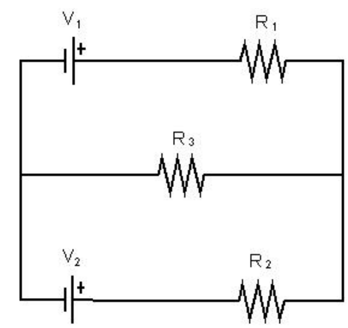


Figure 7.5: Coumpound circuit example

To look at a simple example of this, let's consider a situation with two batteries and three resistors, as shown in figure 7.5. If we let $V_1 = 15$ V, $V_2 = 20$ V, $R_1 = 5.0\ \Omega$, $R_2 = 10\ \Omega$, and $R_3 = 10\ \Omega$, find all the currents.

In this case, we have unknowns (I_1, I_2 and I_3), so we will require 3 equations. We need to define the direction of each current (if we're wrong, we'll just get a negative value for the current, as you will see). Let's say each current travels to the left, as shown. We will also define two loops, loop 1 and loop 2 as indicated by the arrows in figure 7.6.

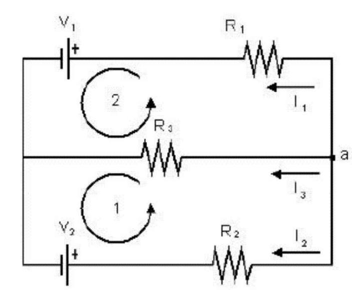


Figure 7.6: Coumpound circuit with currents direction and loops defined

At point a we can see that all currents are leaving the point, so

$$I_1 + I_2 + I_3 = 0 \qquad \text{(eq.1)}$$

From loop 1 we see that

$$-V_2 + I_2R_2 - I_3R_3 = 0$$

or

$$10I_2 - 10I_3 = 20 \qquad \text{(eq.2)}$$

(we subtract the voltages when going the wrong way across a battery, or the correct way across a resistor and vice versa).

From loop 2 we obtain

$$V_1 + I_3R_3 - I_1R_1 = 0$$

or

$$-5I_1 + 10I_3 = -15. \qquad \text{(eq.3)}$$

Putting this in matrix form, we get

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 10 & -10 \\ -5 & 0 & 10 \end{pmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 20 \\ -15 \end{bmatrix}$$

Notice that each line of the first matrix contains the coefficients of the corresponsing equation. When multiplied by the column vector $\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$ we obtain the full equation.

By finding the inverse of the 3×3 matrix and multiplying both sides, we can solve for I_1, I_2 and I_3.

$$\begin{pmatrix} 0.500 & -0.050 & -0.100 \\ 0.250 & 0.075 & 0.050 \\ 0.250 & -0.250 & 0.050 \end{pmatrix} \begin{bmatrix} 0 \\ 20 \\ -15 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0.50 \\ 0.75 \\ -1.25 \end{bmatrix}$$

So $I_1 = 0.50$ A, $I_2 = 0.75$ A and $I_3 = -1.25$ A (i.e. 1.25 A in the opposite direction to that which we defined).

Capacitors in Circuits

Sections 19-6

Capacitors in Parallel

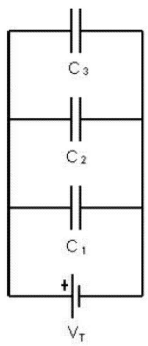


Figure 7.7: Parallel circuit with three capacitors.

When capacitors are connected in parallel, as shown in figure 7.7, each must be charged to a voltage, V, equal to that of the battery (according to Kirchoff's voltage law). The total charge that must leave the battery, then is given by

$$Q = Q_1 + Q_2 + Q_3 = C_1 V + C_2 V + C_3 V = C_{eq} V$$

Where C_{eq} is the equivalent capacitance in the circuit. Thus, $C_{eq} = C_1 + C_2 + C_3$ or generally,

$$C_{eq} = \sum_i C_i. \tag{0.9}$$

Capacitors in Series

If the three capacitors are instead connected in series, we know that if a charge $+Q$ leaves the battery and is deposited on one side of C_1, a charge of $+Q$ flows to one plate of C_3. To maintain neutrality of the conducting wires (which will hold negligible charge compared to the capacitors) each capacitor must have the same charge Q. So

$$V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

Thus $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$, or more generally,

$$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}. \tag{0.10}$$

Steady State RC Circuits

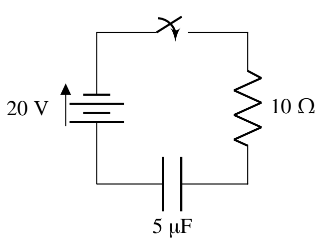


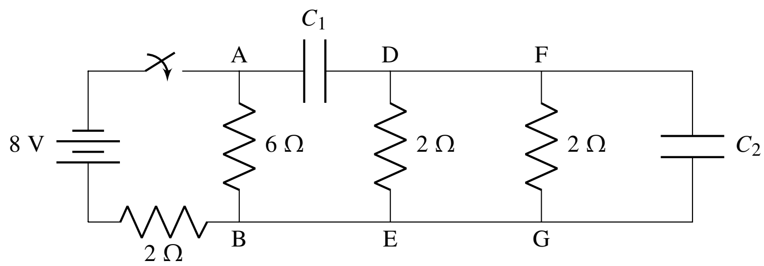
Figure 7.8: Simple RC Circuit.

Let us consider the situation of a circuit with a battery, a switch, a resistor and a capacitor, as shown to the left. After the switch closes, the capacitor will start charging until it reaches full charge. In this process, we will only look at the two extremes: the time at which the switch is first closed ($t = 0$), and the time at which the capacitor reaches full charge ($t = \infty$).

At $t = 0$, the capacitor is uncharged, so the potential difference across the capacitor is 0 V. This means the current is a maximum, $I = \dfrac{V_T}{R} = 2.0$ A. An uncharged capacitor ***has the same effect as a wire***.

At $t = \infty$, the capacitor is fully charged and no current will flow through it. This circuit is said to be in stead state. ***Any branch of an RC circuit in steady state which has a capacitor, will have no current flowing through it.***

Let us consider a more complicated RC circuit as shown below.



1. What is the total current at
   a. $t = 0$?
   b. $t = \infty$?
2. At $t = \infty$, what is voltage across
   a. C_1?
   b. C_2?

1. Let us consider the total currents:
   a. At $t = 0$ the capacitors behave as a wire (or a short circuit), so the potential difference between A and B is zero. So the total current in the circuit is $I_0 = \frac{V}{R} = \frac{8\ \text{V}}{2\ \Omega} = 4$ A.
   b. At $t = \infty$ no current flows through the capacitors (or any branch containing them), so the current flows only through the 6 Ω resistor back to the battery (through the 2 Ω resistor). So $I_\infty = \frac{V}{R} = \frac{8\ \text{V}}{8\ \Omega} = 1$ A.
2. For the voltages across the capacitors, we consider the steady state situations in b. above, in which $V_{AB} = IR = (1\ \text{A})(6\ \Omega) = 6\ \text{V}$. We also know that $V_{DE} = V_{FG} = 0$ since in steady state there is no current passing through these resistors. Therefore,
   a. looking at loop ADEB, we see that $V_{C_1} = 6$ V, and
   b. therefore $V_{C_2} = 0$!