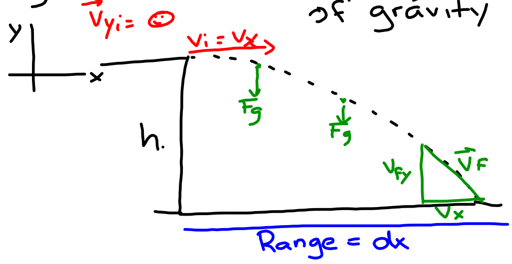
Projectile Motion

Air to Ground Projectiles

Projectile motion - any object under the influence of gravity



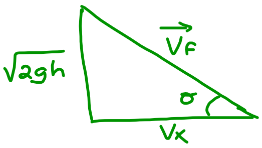
$$\sum \vec{F} = m\vec{a}$$
$$\vec{a} = -g\hat{y}$$

time is y problem
depends only on height

- no acceleration in the x- direction
- acceleration of g ($9.8\,m/s^2$) in the y- direction (down)
- only time crosses the axes

x	y
$V_i = V_x$	$V_i = ☺$
$V_F = V_x$	$a = -g$
$a = ☺$	$d = -h$
$d_x = V_x t$	$d = \cancel{v_i t}^{0} + \frac{1}{2}at^2$
$d_x = V_x \sqrt{\frac{2h}{g}}$	$-h = \frac{1}{2}(-g)t^2$
	rearranging
	$t = \sqrt{\frac{2h}{g}}$
	$V_f^2 = \cancel{V_i^2}^{0} + 2ad$
	$V_f^2 = 2(-g)(-h)$
	$V_f^2 = 2gh$

$V_f = -\sqrt{2gh}$



$$\theta = \tan^{-1}\left(\frac{\sqrt{2gh}}{V_x}\right)$$

$$V_F^2 = V_x^2 + \left(\sqrt{2gh}\right)^2$$
$$V_F = \sqrt{V_x^2 + 2gh}$$

Energy

Top

$PE_i = mgh$

$KE_i = \frac{1}{2}mV_x^2$

Bottom

$PE_f = ☺$

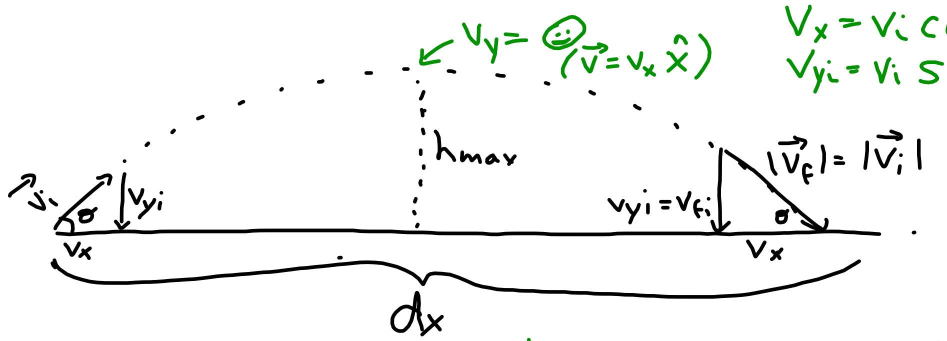
$KE_f = \frac{1}{2}mv_x^2 + mgh = \frac{1}{2}mV_f^2$

$= \frac{1}{2}V_x^2 + gh = \frac{1}{2}V_f^2$

$V_f^2 = V_x^2 + 2gh$

Ground to Ground Projectiles (symmetrical)

$V_x = V_i \cos\theta$
$V_{yi} = V_i \sin\theta$



Time is a y-problem depends only on V_{yi}

X	Y
$d_x = V_x t$	going up

X

$d_x = V_x \dfrac{2V_{yi}}{g}$

$d_x = \dfrac{2V_x V_{yi}}{g}$

$d_x = \dfrac{2(V_i\cos\theta)(V_i \sin\theta)}{g}$

$d_x = \dfrac{V_i^2 \, 2\sin\theta\cos\theta}{g}$

$d_x = \dfrac{V_i^2 \sin 2\theta}{g}$

more explanation coming for this

Y — going up

$V_i = V_{yi}$ $a = -g$ $V_f =$ ☺

$\cancel{V_f}^{☺} = V_i + at$ $\cancel{V_f}^{2☺} = V_i^2 + 2ad$

$V_{yi} = g t_{up}$ ☺ $= V_{yi}^2 + 2(-g)(h_{max})$

$t_{up} = \dfrac{V_{yi}}{g}$ $h_{max} = \dfrac{V_{yi}^2}{2g}$

$V_{yi} = +\sqrt{2gh_{max}}$

going down (same as air to ground)

$V_f = -\sqrt{2gh_{max}}$

sufficient for complete symmetry

$t_{down} = \sqrt{\dfrac{2h_{max}}{g}}$

$t_{down} = \sqrt{\dfrac{2\left(\frac{V_{yi}^2}{2g}\right)}{g}}$ $\dfrac{\frac{?}{g}}{g} = \dfrac{V_{yi}}{g} \times \dfrac{}{g}$ $\sqrt{\dfrac{V_{yi}^2}{g^2}}$

$t_{down} = \dfrac{V_{yi}}{g}$

$t = t_{up} + t_{down} = 2t_{up} = \dfrac{2V_{yi}}{g}$

Remember
$t_{up} = t_{down}$