Bohr Model of the Atom

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Hydrogen Spectrum

visible lines:

λ = 656 nm, 486 nm, 434 nm, 410 nm

Later shown to extend to 365 nm

Balmer

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

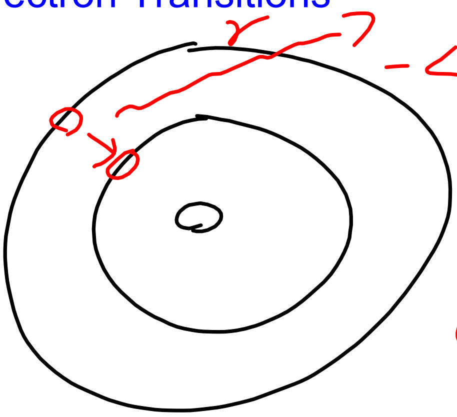
Lyman (UV)

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

Paschen (IR)

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$$

Electron Transitions



$$-\Delta E_e = -(E_\ell - E_u)$$
$$= E_u - E_\ell$$

$$E_\gamma = E_u - E_\ell = hf$$

Bohr

$$L = mvr = n\hbar \qquad \hbar = \frac{h}{2\pi}$$

$$F_{el} = \frac{ke(Ze)}{r^2} = \frac{kZe^2}{r^2}, \quad Z = \# \text{ of protons}$$

$$F_{el} = \frac{ke^2}{r^2}, \quad Z = 1 \text{ for } H$$

$$\frac{ke^2}{r^2} = \frac{mv^2}{r}, \quad v = \frac{n\hbar}{mr}$$

$$\frac{ke^2}{r} = m_e\left(\frac{n\hbar}{m_e r_n}\right)^2$$

$$\frac{ke^2}{\cancel{r_n}} = \frac{\cancel{m_e} n^2 \hbar^2}{m_e^{\cancel{2}} r_n^{\cancel{2}}}$$

$$r_n = \frac{n^2\hbar^2}{m_e k e^2}, \quad \text{For } H,\ n=1$$

$$r_1 = \frac{\hbar^2}{m_e k e^2} = \frac{\left(6.63\times10^{-34}/2\pi\right)^2}{(9.11\times10^{-31})(8.99\times10^{9})(1.602\times10^{-19})^2}$$

$$= 5.29\times10^{-10}\,m$$
$= \text{radius of an H atom}$

Generally,

$$r_n = n^2 r_1$$

Energy

$$KE = \frac{1}{2}mv^2 \qquad E = \frac{1}{2}mv^2 - \frac{ke^2}{r}$$

$$PE = -\frac{ke^2}{r} \qquad E_n = \frac{1}{2}mv_n^2 - \frac{ke^2}{r_n}$$

$$= \frac{1}{2}m\left(\frac{n\hbar}{mr_n}\right)^2 - \frac{ke^2}{r_n}$$

$$\Rightarrow E_n = \frac{\frac{1}{2}\cancel{m}n^2\hbar^2}{m^{\cancel{2}}r_n^2} - \frac{ke^2}{r_n}, \quad r_n = \frac{n^2\hbar^2}{m_e ke^2}$$

$$E_n = \frac{n^2\hbar^2}{2m_e\left(\frac{n^2\hbar^2}{m_e ke^2}\right)^2} - \frac{ke^2}{\left(\frac{n^2\hbar^2}{m_e ke^2}\right)}$$

$$= \frac{n^2\hbar^2}{\frac{2mn^4\hbar^4}{m_e k^2e^2}} - \frac{ke^2}{\frac{n^2\hbar^2}{m_e ke^2}}$$

$$= \frac{\cancel{n^2}\cancel{\hbar^2}m_e k^2e^4}{2\cancel{m_e}n^{\cancel{4}}\hbar^{\cancel{4}}} - \frac{m_e k^2e^4}{n^2\hbar^2}$$

$$= \frac{m_e k^2e^4}{2n^2\hbar^2} - \frac{m_e k^2e^4}{n^2\hbar^2}$$

$$= -\frac{1}{2}\left(\frac{m_e k^2e^4}{n^2\hbar^2}\right)$$

$$E_n = -\frac{1}{n^2}\left(\frac{m_e k^2e^4}{2\hbar^2}\right)$$

$$n=1, \quad E_1 = -\frac{m_e k^2e^4}{2\hbar^2}$$

$$E_1 = \frac{9.11\times10^{-34}(8.99\times10^9)^2(1.602\times10^{-19})^4}{2\left(\frac{6.63\times10^{-34}}{2\pi}\right)^2}$$

$$= -2.17\times10^{-18}\,J = -13.6\,eV$$

This is the energy needed to pull off an e^- from the H atom