Nuclear Decay

While the process of radioactivity is, in fact, random, we can determine the *probability* of decay by looking at macroscopic samples (i.e., samples with large numbers of nuclei). If we assume that each nucleus has the same probability of decaying (undergoing a radioactive process) in each second it exists, then the number of decays, ΔN occurring in a short time Δt is proportional to the time and the number of nuclei, N. It is also reasonable to assume that ΔN will decrease over time (as there will be fewer radioactive nuclei) so

$$\Delta N \propto -N\Delta t$$

or

$$\Delta N = -\lambda N \Delta t$$

$\Rightarrow$ Using Cal.

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

$$\int \frac{1}{N} dN = \int -\lambda \, dt$$

$$\frac{dN}{dt} = -\lambda N$$

$$\ln N + c_1 = -\lambda t + c_2$$

$$\Rightarrow \boxed{N(t) = N_0 e^{-\lambda t}}$$

Since $\frac{\Delta N}{\Delta t} \propto N$ then it must also decrease exponentially in time at the same rate:

$$\frac{\Delta N}{\Delta t} = \left(\frac{\Delta N}{\Delta t}\right)_0 e^{-\boxed{\lambda} t}$$

Defines how radioactive an element is

A note on *e*

1) From the Compound interest formula

2) Taylor Series Expansion

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

3) As the only non-trivial whose derivative is equal to the function itself

$$\frac{d}{dx} e^x = e^x$$

$$\Rightarrow \text{at } x = 0, \ \frac{d}{dx} = 1$$

$\frac{d}{dx} 3^x$ at $x = 1$, Value of 1.10

$\frac{d}{dx} 2^x$ at $x 1$, Value of 0.693

$$(e^x)^{-1} = \ln x$$

Half-Life

Rather than discussing the decay constant, frequently we refer to a radioactive material's half-life ($T_{1/2}$), that is the time, on average, when the number of radioactive nuclei is ½ the original.

$$N(t) = N_0 e^{-\lambda t}$$

t for half-life as $T_{1/2}$

$$N(T_{1/2}) = N_0 e^{-\lambda (T_{1/2})}$$

at $T_{1/2}$, $N(T_{1/2}) = \frac{1}{2} N_0$

$$\frac{1}{2}\cancel{N_0} = \cancel{N_0} e^{-\lambda (T_{1/2})}$$

$$\frac{1}{2} = e^{-\lambda (T_{1/2})}$$

$$\ln\left(\tfrac{1}{2}\right) = -\lambda T_{1/2}$$

$$-\ln(2) = -\lambda T_{1/2}$$

$$\Rightarrow \boxed{T_{1/2} = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda}}$$

units are reciprocal time (s^{-1})

Radioactive Dating

Radioactive carbon-14 makes up 1.3×10^{-10} % of all naturally occurring carbon. As long as a plant or animal is alive it continually replenishes its store of carbon-14 by ingestion. Once dead, however, the amount of carbon-14 decreases by beta decay to nitrogen-14:

$$^{14}_{6}C \rightarrow {}^{14}_{7}N + \beta^{-} + \bar{\nu}$$

Carbon-14 has a half-life of 5730 years.

If we find a bone fragment that has 200 g of carbon content and registers 16 decays/s, what is its age?

1) Determine N_0 of carbon-14 (recall $N_A = 6.02 \times 10^{23}$ atoms/mol)

Assume it is (nearly) all ^{12}C

$$n_{^{12}C} = \frac{m_{^{12}C}}{M_{^{12}C}} = \frac{200\,g}{12.5\,g/mol} = 16.7\,mol$$

$$N_{^{12}C} = (16.7\,mol)\left(6.02 \times 10^{23}\,\frac{nucl.}{mol}\right)$$
$$= 1.0 \times 10^{25}\,nucl.$$

If we assume % ^{14}C is 1.3×10^{-10} %

$$N_{0,^{14}C} = N_{^{12}C} \times (1.3 \times 10^{-10})$$
$$= 1.3 \times 10^{13}\,nucl.$$

2) Determine $\left(\frac{\Delta N}{\Delta t}\right)_0$

For C-14, $\lambda = 3.83 \times 10^{-12}\ s^{-1}$

Since $\lambda = \frac{\ln 2}{T_{1/2}}$

$$\left(\frac{\Delta N}{\Delta t}\right)_0 = -\lambda N_0$$

$= -3.83 \times 10^{-12}\ s^{-1}\ (1.3 \times 10^{3}\ \text{nuclei})$

$= -5.0\ \frac{\text{nuclei}}{s} = 5.0\ \frac{\text{decays}}{s}$

3) Use the exponential decay equation to determine *t*.

current decay rate → initial rate of decay

$$\frac{\Delta N}{\Delta t} = \left(\frac{\Delta N}{\Delta t}\right)_0 e^{-\lambda t}$$

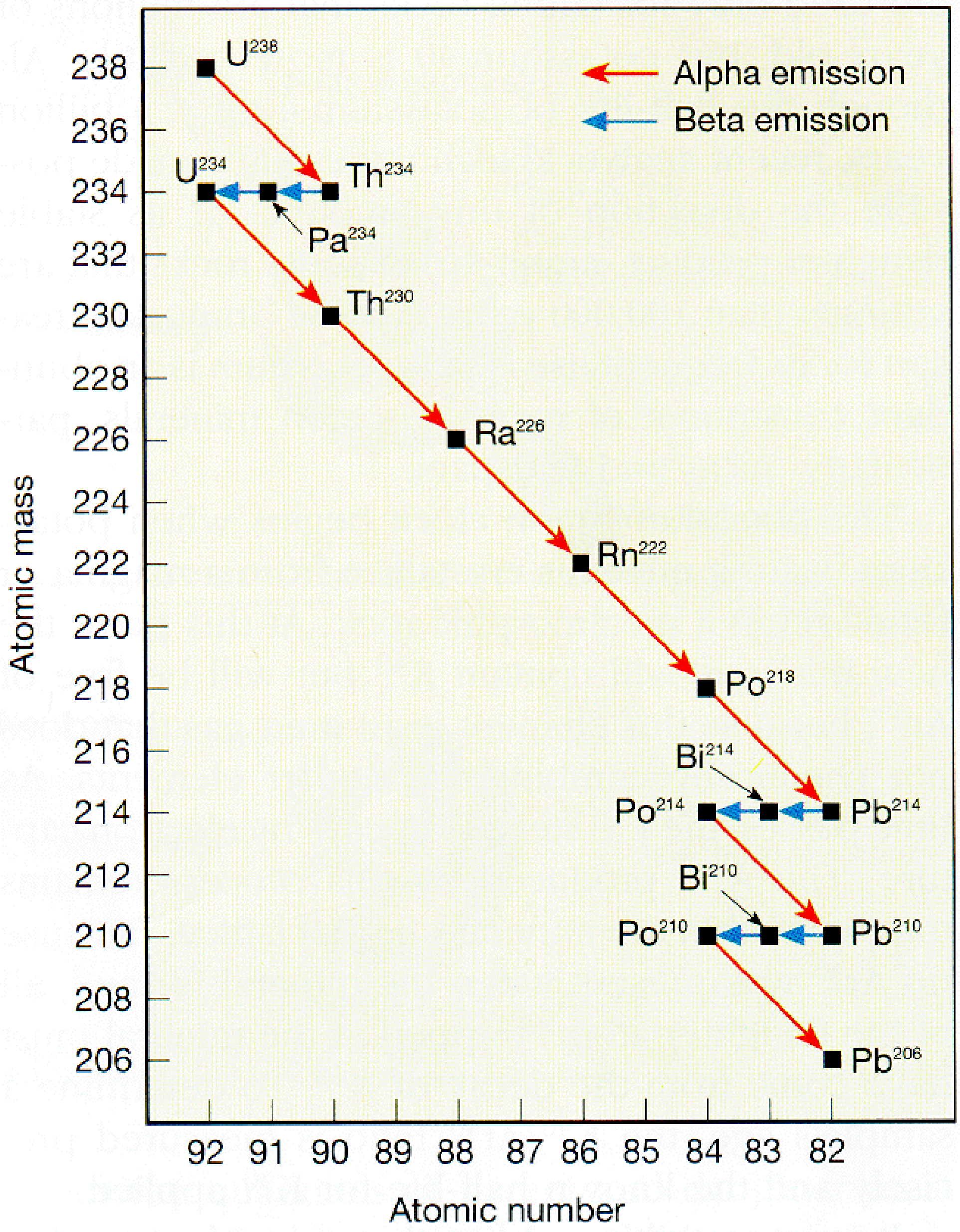
$$16 \frac{\text{decays}}{s} = 50\, e^{-\frac{\ln 2}{5730\,y} t}$$

$$\frac{50}{16} = e^{\frac{\ln 2}{5730\,y} t} \Rightarrow \ln\left(\frac{50}{16}\right) = \frac{\ln 2}{5730\,y} t$$

$$\Rightarrow t = \frac{\ln\left(\frac{50}{16}\right) \cdot 5730\,y}{\ln 2} = 9419\,y$$

$$\simeq \boxed{9400\,y}$$

Radioactive Series - Uranium-238 to Lead-206



Another example - same series