Chapter I

Electromagnetism

Day 1 Magnetism

Sections 20-1, 20-13

An investigation of *permanent magnets* shows that they only attract certain metals – specifically those containing iron, or a few other materials, such as cobalt or nickel. While they seem to have properties similar to static electricity, they are much more limited in the items they attract.

Permanent magnets have two poles. We label these North and South. Like with electric charges, these definitions are arbitrary and were originally defined based on the direction the pole would point. The North on the compass points toward, not the North pole, but what we call the *magnetic North pole*. However, since we know that the North pole on a compass is attracted to a negative pole (opposites attract and likes repel), this tells us that what we call magnetic North is in fact a magnetic *South* pole.

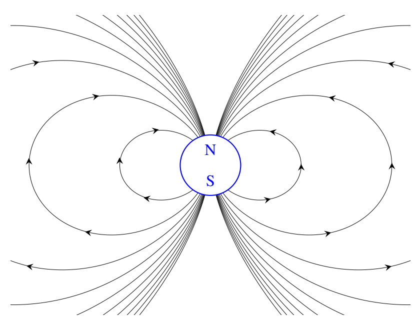


Figure 1.1: A schematic diagram of Earth's magnetic field.

There are similarities between magnets and static charges. Likes attract, opposites repel. They have analogous fields - the electric field between two charges looks like the magnetic field between two poles. We can use a compass or temporary magnets (such as iron filings) to observe and investigate these fields. We should note, however that there are significant differences. While we readily observe independent + and – static charges, we have not yet observed a single magnetic North without a corresponding South pair. That is, we have not yet observed a *magnetic monopole*. Quantum field theories, however, do predict the existence of magnetic monopoles, and many scientists are searching for their existence. It is believed that they may have existed in an earlier stage of the universe.

What causes magnetism? It is believed to be related to electron "spin" and the ability of a material to have its electron spins aligned. Electron spin was first thought of as electron spinning on its axis (like earth) while rotating around the nucleus (like the sun). This is oversimplified as it is difficult to view an electron as an actual localized object (Heisenberg uncertainty principle) much less a spinning one.

Other topics of discussion:

Domains, aligned vs. random domains.
Magnetic field, B, SI unit Tesla (T).
Investigate B between two large surfaces (analogous to E between two plates).

Day 2 Electric Currents and Magnetism

Sections 20-2 to 20-4

In 1820, Hans Christian Oersted discovered a connection between magnetism and electric currents. He found that a compass needle was influenced by an electric current in a wire.

If the wire is straight, the magnetic field lines are concentric circles around the wire.

If the wire is circular (or if there are many loops like a solenoid), the magnetic field is similar to that produced by a bar magnet.

The direction of these magnetic field lines can be determined using the 1st RH rule. *If you point your thumb in the direction of the current and wrap your fingers around the wire, your fingers will indicate the direction of the magnetic field.*

If a wire carrying a current can produce a magnetic field, and this wire is placed in a magnetic field, we would expect the two fields to interact much like two electric fields interact (i.e., there will be a force)

Experimentally, we find that the force is perpendicular to both the direction of the current and the direction of the magnetic field, and its direction can be determined by a 2nd RH rule. *If your thumb points in the direction of the current, and your fingers point in the direction of the magnetic field, your palm will point in the direction of the force.*

We find from careful experiment that $F \propto \ell IB \sin\theta$, where B is the magnetic field strength, ℓ is the length of the current carrying wire **in the magnetic field**. I is the current, and θ is the angle between B and I. The magnetic field, B, is defined so that the proportionality constant in the above relationship is 1, giving

$$F = \ell IB \sin\theta \tag{1.1}$$

or more properly $F = \ell I \times B$ (where $\times$ represents the vector cross product, but the first equation will do fine for now).

Notations:

$\otimes$ is a vector pointing into the page (looking at the tail feathers of an arrow)
$\odot$ is a vector pointing out of the page (looking at the tip of an arrow).

If we consider a moving, positive charge, q then :
$I = \frac{q}{t}$ and $F_{max} = \frac{\ell qB}{t}$ but $\frac{\ell}{t} = v$, so $F_{max} = qvB$ or

$$F = qvB \sin\theta. \tag{1.2}$$

We'll look at some sample problems next day.

Day 3 Electric Currents and Magnetism - cont'd

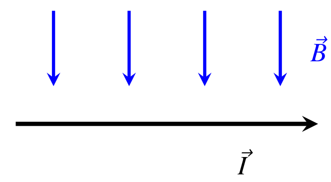
Sections 20-2 to 20-4

Sample problems:

1. A current carrying wire perpendicular to the magnetic field

A wire of length 20 cm carrying a current of 5.0 A is placed in and perpendicular to a magnetic field of strength 1.5 T, as shown below. What is the force acting on the wire (magnitude and direction)?

$$\begin{aligned} F &= \ell I B \sin\theta \\ &= (0.20 \text{ m})\,(5.0 \text{ A})\,(1.5 \text{ T}) \\ &= 1.5 \text{ N} \otimes \end{aligned}$$



2. Current carrying wire, not perpendicular to magnetic field.

Same question as above, change direction of current, at angle of 60°.

3. Electron moving through a magnetic field.

An electron traveling at $4.5x10^6$ m/s enters a magnetic field of strength $B = 0.75$ T perpendicular to the field.
a) Describe the path of the electron.
b) What force is acting on the electron?
c) What is the radius of the electron's motion?

4. Proton moving through a magnetic field.

Same question as above except using a proton instead of an electron.

5. Electron moving through a magnetic field enters at a non-90° angle to the field.

Same question as 3, but angle is 60°.

Day 4 Magnetic Fields and Wires

Sections 20-5 to 20-7

Magnetic Field from a Long Wire

Let us consider the magnetic field caused by a "long" wire. What does long mean? From a theoretical point of view, we are speaking of an infinitely long wire (i.e., $\ell = \infty$, however, from a practical point of view, we simply mean that the length is much greater than the distance from the wire that we are investigating the field (i.e., $\ell \gg r$).

From experiment we can determine that the magnetic field, B is proportional to the current and inversely proportional to the distance from the wire (i.e., $B \propto \frac{I}{r}$). This may seem a bit strange as we are used to inverse square laws, however from a physical point of view, in those cases, we were considering the distance from a point (or a zero-dimensional object). Here we are considering the distance from a uniform line (or a 1-dimensional object).

We define the constant μ_0 as the *permeability of free space*: $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$. The proportionality constant for the above relationship is *defined as* $\frac{\mu_0}{2\pi}$. Note that this has implications on our definition of current. This means our definition of current is in fact tied to our definition of B, and therefore our definition of the coulomb is tied to B. This gives us

$$B = \frac{\mu_0 I}{2\pi r}. \tag{1.3}$$

The Force between Two Parallel Wires

Knowing $F = \ell I B$, then the force per unit length on a current carrying wire from another parallel current carrying wire is found by combining equations 1.1 and 1.3. If we consider the two wires to be carrying currents of I_1 and I_2 respectively, we get:

$$\frac{F}{\ell} = I_1 \left(\frac{\mu_0 I_2}{2\pi r} \right)$$

or

$$\frac{F}{\ell} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{r}. \tag{1.4}$$

We will look at examples involving suspending one wire with another and discussing stable versus unstable equilibria.

Day 5 Cylindrical Coordinate Systems and Ampere's Law

Section 20-8

Cylindrical Coordinate Systems

We will spend considerable time discussing a cylindrical coordinate system, with r, z, ϕ as opposed to the Cartesian coordinate system x, y, z and vectors and unit vectors in each. We will also discuss advantages of using different coordinate systems.

Ampere's Law

Ampere's law discusses the relationship between the current in any shaped wire with the magnetic field that it produces. In its discrete form (non-calculus based form) it says

$$\sum B_{\parallel} \Delta\ell = \mu_0 I_{in} \tag{1.5}$$

where $B_{\parallel}$ is the magnetic field parallel to the section of a closed loop, $\Delta\ell$ and I_{in} is the current enclosed by the (arbitrary) loop. In integral form, it (more accurately) looks like this:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{in}$$

where $\oint$ represents an integral over a closed path or loop and $\vec{B} \cdot d\vec{\ell}$ is the dot product between the magnetic field and the path.

Examples:

1. Simple case – long wire carrying current I.

   By symmetry, we realize that the magnetic field cannot be dependent on z (up and down the length of the wire), because the wire looks the same everywhere. Similarly, we realize it cannot depend upon the angle around the wire (the wire looks the same from any angle around it). Therefore the magnetic field depends solely on the radius, r and the current I. We choose a circular path around the wire of radius r to investigate the field. We realize the magnetic field must be constant at this radius, so

   $\sum B_{\parallel} \Delta\ell = B\,(2\pi r) = \mu_0 I_{in}$.

   Solving for B we get $B = \dfrac{\mu_0 I}{2\pi r}$ like we had before.

2. Long solenoid with current I, radius R and N loops of wire per unit length.

   In this case, we again realize, by symmetry, that the magnetic field is dependent on R, and not the position along or around the solenoid. We can choose several paths to investigate the magnetic field.

   a. circular path inside the solenoid ($B_\phi = 0$, for $R < r$)
   b. circular path outside the solenoid ($B_\phi = \dfrac{\mu_0 I}{2\pi r}$, for $R > r$)
   c. rectangular path outside the solenoid. ($B_z = 0$, for $R > r$)
   d. rectangular path across the edge of the solenoid: $I_i n = N\ell I$
      So, $B_z \ell = \mu_0 N \ell I$ giving $B_z = \mu_0 N I$ for $R < r$.

Day 6 Torque on a Loop and Mass Spectrometers

Sections 20-9, 20-12

Torque on a Loop

Consider a rectangular loop of wire height a, width b, placed in a uniform magnetic field parallel to the plane of the loop.

$F_{left} = B\ell I = BIa\otimes$
$F_{right} = B\ell I = BIa\odot$

Since there is no force due to the top and bottom wires of the loop (because they are parallel with the field) we get
$\tau_{left} = Fr_{\perp} = BIa\left(\frac{b}{2}\right)$ clockwise (when viewed from the top of the loop) and
$\tau_{right} = Fr_{\perp} = BIa\left(\frac{b}{2}\right)$ clockwise so

$$\sum \tau = BIab = BIA$$

where A is the area of the loop. It turns out that this is generally true regardless of the shape of the loop. If we have, instead of 1 loop, N loops we have

$$\sum \tau = BNIA. \tag{1.6}$$

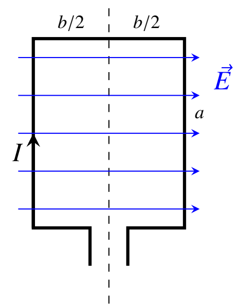


Figure 1.2: Torque on a rectangular loop of wire

The magnetic dipole moment, M is given by

$$M = NIA \tag{1.7}$$

and is defined as being perpendicular to the area using a right hand rule (wrap fingers in the direction of the current, the thumb points in the direction of M).

This is the fundamental principle behind motors (a device which turns electrical energy into rotational energy).

Mass Spectrometers

A charged particle, q, can be accelerated from rest by a voltage V. At this point, it has kinetic energy $KE = qV = \frac{1}{2}mv^2$. When it enters a constant magnetic field B, it will travel in a circular path of radius r, where

$$F = qvB = \frac{mv^2}{r}.$$

If v is constant because of a velocity selector (see text pp. 607 – we can do this with electric and magnetic fields), then m/r is a constant. So, a larger mass will have a larger radius of curvature. By calibrating the machine with a known isotope, we can determine the other isotopes by their radii of curvature.

$$m = \frac{qBr}{v} \tag{1.8}$$