Chapter II

Electromagnetic Induction

Day 1 Induced EMF, Faraday's Law and Lenz's Law

Sections 21-1 to 21-2

Electromotive Force

Electromotive force (EMF or $\mathcal{E}$) is a misnomer, as it is not really a force but a potential difference (voltage). It is defined as the potential difference between the terminals of a battery when no current flows. When a current flows, the electromotive force is determined by the sum of the terminal voltage and the voltage drop *within the battery*

$$\mathcal{E} = V_{term} + V_r. \tag{2.1}$$

With the discovery that electric currents produced magnetic fields, came the question "Can magnetic fields produce electric currents?" It was discovered independently by Joseph Henry and Michael Faraday (c. 1830-31) that they could.

In a setup like the one to the right, Faraday used a battery to make a magnetic field (at X), causing the iron ring to become magnetized. However, he observed that there was no current at Y *except when* the switch was being opened or closed.

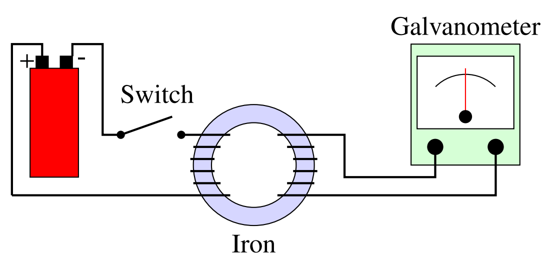

Figure 2.1: Faraday's experiment to induce an EMF.

The conclusion that he drew was that if the magnetic field were steady, there would be no current, however, if the magnetic field changes, a current is produced. This is called induction.

We define the *magnetic flux*, Φ_B to be

$$\Phi_B = B_{\parallel}A = BA\cos\theta \tag{2.2}$$

where θ is the angle between B and A (note that the direction of a surface is perpendicular to the surface). It is proportional to the number of field lines passing through the loop. The unit of flux is the Weber (1 Wb = 1 T·m^2).

Faraday's Law of Induction and Lenz's Law

Faraday's law of induction is the basic law of electromagnetism, relating the induced EMF to the changing magnetic flux.

$$\mathcal{E} = -N\frac{\Delta\Phi_B}{\Delta t} \tag{2.3}$$

or more properly, in calculus form

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}.$$

In each case, N represents the number of loops of wire surrounding an area. The negative sign indicates the direction of the change: a decreasing flux causes a positive $\mathcal{E}$ whereas an increasing flux causes a negative $\mathcal{E}$.

Lenz's law simply states "*an induced $\mathcal{E}$ gives rise to a current whose magnetic field opposes the original flux.*" If this were not the case, and the induced $\mathcal{E}$ gave rise to a current which supported the original flux, then this current would cause an increase in B (and therefore flux) which would cause an increase in the induced current, which would in turn increase B. It would essentially become an unlimited power source, which would violate just about every law of energy and thermodynamics in the universe. . .

Day 2 Ways of Inducing $\mathcal{E}$

Sections 21-3 to 21-4

Since $\mathcal{E} = -N\dfrac{\Delta\Phi_B}{\Delta t}$, and $\Phi_B = B_{\parallel}A$, there are three possible ways of inducing EMF.

1. Change the size of the magnetic field.

Example: A single circular wire loop of radius 10 cm placed in and perpendicular to a magnetic field that increases from 5.50×10^{-5} T to 3.00×10^{-4} T in 2.00 s. If the resistance of the wire is 1000 Ω/m, find the current induced in the wire.

$A = \pi r^2 = 0.0314\ \text{m}^2$
$\Delta\Phi_B = \Delta BA = \left(3.00 \times 10^{-4} - 0.550 \times 10^{-4}\ \text{T}\right)\left(0.0314\ \text{m}^2\right) = 7.69 \times 10^{-6}\ \text{Wb}$

$$\mathcal{E} = -1\frac{7.69 \times 10^{-6}\ \text{Wb}}{2.00\ \text{s}} = 3.85 \times 10^{-6}\ \text{V}$$

Since $\mathcal{E} = IR$

$$I = \frac{3.85 \times 10^{-6}\ \text{V}}{2\pi\,(0.10\ \text{m})\,(1000\ \Omega/\text{m})} = 6.13\ \text{nA}$$

which is why we don't observe this effect with ordinary sized magnetic fields!

2. Change the area

Example: A wire moving along the open end of a U-shaped loop

Consider a constant magnetic field, B. If the U is of width ℓ and the wire is moving with speed v. Then

$\Delta A = \ell v \Delta t$, so

$$\mathcal{E} = -\frac{B\ell v\Delta t}{\Delta t} = -B\ell v.$$

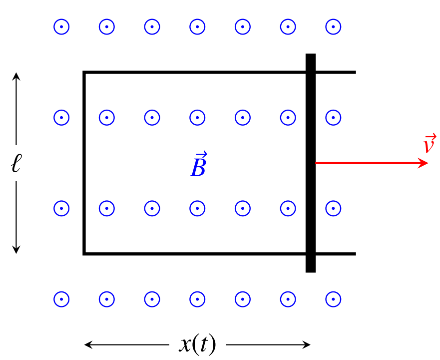

Figure 2.2: U-shaped loop with moving wire.

3. Change the angle (i.e., an electric generator - a topic for the next lesson)

A note on example # 2. If the moving wire is not touching the U conductor, electrons will be forced up the wire in order to fulfill Faraday's law. This means they are experiencing a force. Since electric field, E, is defined as $E = \dfrac{F}{q}$, this means a change in magnetic flux causes an electric field. Since $E = \dfrac{F}{q}$, and that force $F = qvB$, then the induced electric field is

$$E = vB \tag{2.4}$$

Day 3 Generators $\mathcal{E}$

Sections 21-4 to 21-5

A generator is probably one of the greatest consequences of Faraday's law. A generator is essentially the reverse of a motor. We can produce electrical energy from rotational kinetic energy. Consider a rectangular loop of height, h, and length, ℓ, placed inside a uniform magnetic field B. The loop is rotated around the axis parallel to the length, ℓ, at a radius of $h/2$. Then

$\mathcal{E} = NB\ell v \sin\theta$,

where θ is the angle between the magnetic field and the velocity of the loop. (That is, when the direction of the area is parallel to the magnetic field, the change in magnetic flux is 0.)

Since $\mathcal{E}$ is the same in the other side of the loop (the other wire of length ℓ) the EMF's add together, effectively doubling the EMF:

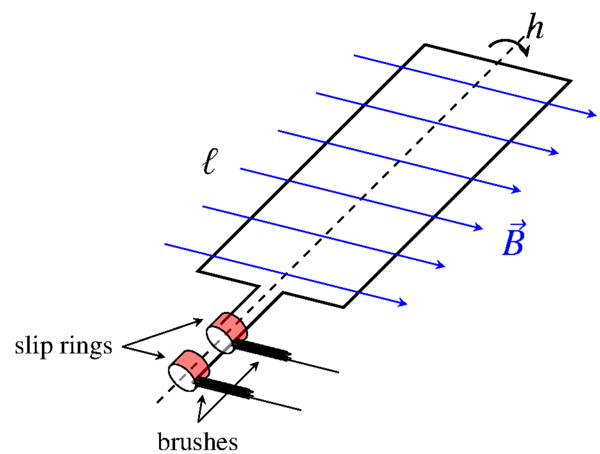

Figure 2.3: A Simple AC Generator

$\mathcal{E} = 2NB\ell v \sin\theta$

but $\theta = \omega t$ and $v = \omega r$ and the radius of the loop is $h/2$, so

$\mathcal{E} = \frac{2NB\ell h\omega \sin\omega t}{2}$, but $\ell h = A$ (area) so

$$\mathcal{E} = NBA\omega \sin\omega t. \tag{2.5}$$

Note: if $\omega t = 2\pi$, then it has completed rotation, so $t = T$ and $T = 1/f$. Thus $\omega = 2\pi f$, and $f = 60$ Hz in North America).

Day 4 Resistivity, Root Mean Square, Transformers

Sections 21-4 to 21-7

Resistivity

We know that the resistance of a wire is directly proportional to the length, ℓ, of the wire and inversely proportional to the cross-sectional area, A. Thus

$$R \propto \frac{\ell}{A}, \text{ or R = constant} \times \frac{\ell}{\text{A}}.$$

This proportionality constant is known as the resistivity, ρ, of a substance, and is dependent upon the substance and the temperature (we know that as temperature increases, resistance also increases). Page 534 in your text has several resistivities listed and the temperature coefficients.

$$R = \rho\left(T\right)\frac{\ell}{A} \tag{2.6}$$

where

$$\rho\left(T\right) \approx \rho_0\left(1 + \alpha T\right) \tag{2.7}$$

Root Mean Square (rms)

Root-mean-square is the square root of the average of the square of a function. This is particularly useful for discussing the average output of a function, namely in our case, a sinusoidal function like sin or cos. It is also used, for example, in the calculation of many statistical items, such as average deviation (which is essentially the rms of the variance from the mean).

To find the root mean square of a sinusoidal function:

$\sin\theta$

Square it: $\sin^2\theta$

Average it over all θ: We know that $\sin^2\theta + \cos^2\theta = 1$, and that the two are essentially the same functions with a 90° phase change, so on average each has a value of $\frac{1}{2}$.

Take the square root: $(\sin\theta)_{rms} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

Transformers

A transformer is a device for increasing or decreasing ac current. You will find transformers along most power lines (energy is lost from power lines at a rate proportional to the square of the current, so it is beneficial to run high voltage-low current lines) as well as many household devices (e.g. portable phones, battery chargers, battery eliminators, etc.). This works by having an iron core with two coils of wire wrapped around it, known as a primary coil and a secondary coil (see diagram to the right).

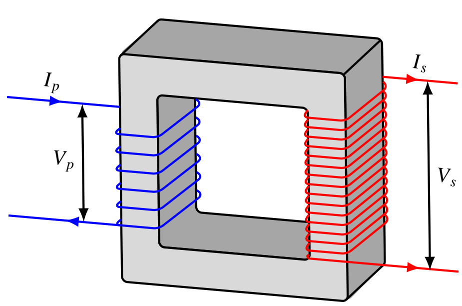

Figure 2.4: A step up transformer with $N_P = 6$ and $N_S = 16$.

Transformers are designed so that nearly all the magnetic flux from the primary coil passes through the secondary coil, and that energy losses are minimized (most transformers are highly efficient – up to 99% or more). For our purposes we will assume the ideal.

The induced voltage in the secondary coil, then, is

$$V_S = -N_S \frac{\Delta\Phi_B}{\Delta t}.$$

This, however, causes an opposing magnetic flux to the original magnetic flux, which is equal and opposite:

$$V_P = -N_P \frac{\Delta\Phi_B}{\Delta t}.$$

Note this is **only** true for ac current (dc current produces no flux). Dividing these two equations we get

$$\frac{V_P}{V_S} = \frac{N_P}{N_S} \tag{2.8}$$

When $N_P > N_S$ it is called a step-down transformer and when $N_P < N_S$ it is called a step-up transformer.

While it may seem we are getting something for nothing, this is not the case, since $P = IV$, by conservation of energy, $P_P = P_S$ (assuming 100% efficiency), so

$$\frac{I_P}{I_S} = \frac{N_S}{N_P}. \tag{2.9}$$

So if the voltage "steps-up" the current must drop proportionally. Since power leakage occurs at a rate $P = I^2R$ (where R is the resistance of the carrying wire), it is beneficial to run a low current and a high voltage to minimize power loss.

Example: Generator used to power device at 220 V, 100 A over a distance of 25.0 km (assume resistance of 0.100 Ω/km) without and with transformers ($N_S/N_P = 1000$ step-up and $N_P/N_S = 1000$ step down). Determine power loss and voltage required by the generator.

Day 5 Mutual and Self Inductance, Energy in Magnetic Field

Sections 21-9 to 21-10

Mutual Inductance

If two coils of wire are placed near one another (such that the magnetic flux caused by one coil is related to the magnetic flux passing through the other coil), then a changing current in the first coil will cause an induced EMF in the second. Since the flux is proportional to the current (recall $B = \frac{\mu_0 I}{2\pi r}$, and the areas are constant) then the EMF must be proportional to the change of current in the first coil:

$$\mathcal{E}_2 = -M\frac{\Delta I_1}{\Delta t}, \tag{2.10}$$

where M is a proportionality constant called the mutual inductance. Note the negative sign is because the EMF must oppose the change in current. The units of M are Vs/A or Ω·s called the henry (H).

Self Inductance

The same situation is true for a single coil of wire in which the current is changing. If a current is increasing, then there is an increase of magnetic flux produced. This produces a back EMF which opposes the change. This property is called self inductance, and similar to mutual inductance is given by:

$$\mathcal{E}_1 = -L\frac{\Delta I_1}{\Delta t}, \tag{2.11}$$

where L represents the self-inductance.

Inductors are beneficial in many electronic devices, most notably in analog tuners (such as a radio) or in surge protectors.

It can be shown (see example 21-11) that the self-inductance in a solenoid is given by:

$$L = \frac{\mu_0 N^2 A}{\ell}. \tag{2.12}$$

Energy Stored in a Magnetic Field

Similar to the energy stored in a capacitor, the energy stored in an inductance L, carrying a current I is given by

$$U = \frac{1}{2}LI^2. \tag{2.13}$$

The energy density (energy per unit volume) stored in a magnetic field can be shown to be

$$u = \frac{1}{2}\frac{B^2}{\mu_0} \tag{2.14}$$

for a solenoid, but is valid for any region of space in which a magnetic field exists.