Chapter IV

Nuclear Physics and Chemistry

Day 1 Nuclear Structure and Binding Energy

Sections 30-1 to 30-2

Nuclear Structure

The nucleus is made of protons and neutrons. The proton (p^+) is positively charged with a rest mass of $m_p = 1.6727 \times 10^{-27}$ kg. The neutron was discovered in 1932 by James Chadwick. The neutron (n^0) is electrically neutral with a rest mass $m_n = 1.6749 \times 10^{-27}$ kg.

We represent a nucleus in the form ${}^{A}_{Z}\text{X}$, where A is the atomic mass number, Z is the atomic number and X is the chemical symbol, e.g. ${}^{4}_{2}\text{He}$ would be Helium-4.

Recall that $A = Z + \#n^0$.

Isotopes are nuclei that have the same number of protons, but a different atomic mass number. Carbon, for example has 6 isotopes (A=11-16). Of these isotopes, ${}^{12}\text{C}$ is most abundant (about 98.9%) and ${}^{14}\text{C}$ makes up about 1.1% of naturally occurring carbon.

The size of nuclei can be estimated using the approximation

$$r \approx \left(1.2 \times 10^{-15}\right) A^{1/3}. \tag{4.1}$$

Atomic Mass Units

For convenience, nuclear masses are often given in unified atomic mass units (u), also called simply atomic mass units.

On this scale ${}^{12}_{6}\text{C}$ has a mass of 12.000 000 u. Other nuclei are compared to the mass of hydrogen.

	Mass		
Object	kg	u	MeV/c^2
Electron	9.1094×10^{-31}	0.00054858	0.51100
Proton	1.67262×10^{-27}	1.007276	938.27
${}^{1}_{1}\text{H}$	1.67353×10^{-27}	1.007825	938.78
Neutron	1.67493×10^{-27}	1.008665	939.57

Notice that masses for atoms are given for electrically neutral atoms (hence atomic mass NOT nuclear mass). However, when dealing with nuclear equations, we can use atomic masses as long as we make sure that the number of electrons are balanced.

Since $E = mc^2$, $m = E/c^2$, and c is a constant, the mass is often given in this form for the purpose of calculating energy (i.e. in MeV/c^2). 1u = 931.5 MeV/c^2.

Binding Energy

If we look at $^{12}_{6}$C, it has the equivalent of 6^1_1H + $6n^0$. From a mass point of view, this is

6 (1.007825 u + 1.008665 u) = 12.09894 u.

Carbon-12, however, has a mass of 12.000000 u.

Δm = 12.09894 u – 12.000000 u = 0.09894 u.

Where did this mass go? By Einstein's equivalence of mass and energy, it has obviously gone to energy (in the form of radiation, KE of particles, etc.) This difference in mass is also known as the **mass defect**. The energy given by

$$BE = \Delta mc^2 \tag{4.2}$$

is known as the **total binding energy**.

The total binding energy for carbon, therefore is

$$BE = \Delta mc^2 = (0.09894 \text{ u})x931.5 \text{ MeV/u} = 92.16 \text{ MeV}.$$

The average binding energy per nucleon is given by

$$BE_{ave} = \frac{\Delta mc^2}{A}. \tag{4.3}$$

To find the binding energy of the last neutron (see appendix F for isotopes), we compare the atomic masses before and after the neutron is removed. For example we can find the mass defect and hence the BE for the nuclear equation

$$^2_1\text{H} \rightarrow {}^1_1\text{H} + {}^1_0\text{n}.$$

Notice: Nuclei are made up of protons and neutrons, and therefore are electrically repulsive. Since the gravitational force is much smaller than the electrical force, we must conclude that there is another force holding the nuclei together. This force is called the strong nuclear force which acts at very short distances (effective radius is of the order of 0.1 fm to 1 fm). There is a fourth force, the weak nuclear force which also acts at small distances, but as the name implies, this is also weak, and is only visible in certain radioactive decay processes.

Day 2 Nuclear Decay

Sections 30-3 to 30-6

Radioactive decay occurs in atoms where the force of electrical repulsion in the nucleus is greater than the attractive force of the strong nuclear force. In this process, the nucleus essentially changes in one of 5 ways, resulting in a changed nucleus, known as a ***daughter nucleus***. (The original nucleus is known as a ***parent nucleus***). This changing of one element into another is known as ***transmutation***.

Alpha Decay

The first form of decay is alpha ($\alpha-$) decay. In this process, the nucleus emits an α particle, which is essentially the nucleus of a helium atom ($^4_2$He).

α decay can be written as

$$^A_Z\mathrm{N} \rightarrow {}^{A-4}_{Z-2}\mathrm{N}' + {}^4_2\mathrm{He} \tag{4.4}$$

where N is the parent, N′ is the daughter and A and Z are the atomic mass and number of the parent nucleus.

The energy released in this process is called the ***disintegration energy***, Q.

$$Q = \left(M_p - M_D - m_\alpha\right)c^2 \tag{4.5}$$

If $Q < 0$, the decay cannot occur. The disintegration energy is usually translated into KE of the daughter nucleus and the alpha particle (which proportions can be determined using the conservation of energy).

Beta (β^-) Decay

In beta β^- decay, the nucleus emits an electron when it decays (this electron is called a β^- particle). It should be noted that this β^- particle is not an orbital electron, but is created within the nucleus. In the process, a neutron disintegrates into a proton, electron and a neutrino. The neutrino was a particle proposed by Wolfgang Pauli in 1930, due to the fact that the majority of electrons emitted from the nucleus in this process did not have enough KE to meet conservation of energy laws. The neutrino (ν), therefore carries momentum and energy and a very tiny rest mass (in fact it was believed to be massless until recently). In fact, it is an antineutrino ($\overline{\nu}$) produced in β^- decay (this is related to the changes in the quarks, which we will not concern ourselves with).

β^- decay therefore can be represented by

$$^A_Z\mathrm{N} \rightarrow {}_{Z+1}^{\;\;A}\mathrm{N}' + e^- + \overline{\nu}. \tag{4.6}$$

Nuclei which undergo β^- decay are nuclei which have too many neutrons for the number of protons (e.g. $^{14}_6$C).

Positron (β^+) Decay

Like β^- decay, β^+ decay occurs by the emission of a positively charged electron (known as a positron, e^+) The positron is the antiparticle of the electron, because it has the same properties as an electron but opposite charge (and some other quantum conditions). These are nuclei which have too few neutrons for the number of protons (e.g.$^{19}_{10}$Ne). In general

$$ {}^{A}_{Z}\mathrm{N} \to {}_{Z-1}^{\ \ A}\mathrm{N}' + \beta^+ + \nu. \tag{4.7}$$

Electron Capture (K-Capture)

In this case, an electron is absorbed into the nucleus (usually from the innermost K-shell, hence K-capture). In this case, the daughter nucleus is produced with a neutrino. It is inferred by the detection of X-rays as the electrons jump down to fill the empty shell and the production of new nuclei. In general

$$ {}^{A}_{Z}\mathrm{N} + e^- \to {}_{Z-1}^{\ \ A}\mathrm{N}' + \nu. \tag{4.8}$$

Note: In all β decay the neutrino interacts with matter only through the weak nuclear force.

Gamma (γ) Decay

Gamma decay occurs when a nucleus is in an excited state (it could arrive in this state by some previous decay). This excited state is said to be a ***metastable state*** and the nucleus is said to be an isomer. In this case, the nucleus drops to a lower state (or ground state) and emits a γ-ray.

Note: While γ-rays typically have higher energy than an X-ray, their energy ranges do overlap. However, if the photon is produced from an electron-atom interaction, we call it an X-ray, but if it occurs from a nuclear interaction we call it a γ-ray.

In general

$$ {}^{A}_{Z}\mathrm{N}^* \to {}^{A}_{Z}\mathrm{N} + \gamma \tag{4.9}$$

where N* is the metastable state.

Day 3 Half-Life, Radioactive Dating and Radioactive Series

Sections 30-7 to 30-11

Conservation of Nucleon Number

What we have seen to this point are nuclear equations (e.g. ${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$ is an example of α-decay). You will notice that in all equations the number of nucleons is the same on each side of the reaction. This is true in all nuclear reactions and is known as the ***law of conservation of nucleon number***.

Nuclear Decay

While the process of radioactivity is, in fact, random, we can determine the probability of decay by looking at macroscopic samples (i.e., samples with large numbers of nuclei). If we assume that each nucleus has the same probability of decaying (undergoing a radioactive process) in each second it exists, then the number of decays, ΔN occurring in a short time Δt is proportional to the time and the number of nuclei, N. It is also reasonable to assume that ΔN will decrease over time (as there will be less radioactive nuclei) so

$$\Delta N = -\lambda N \Delta t, \tag{4.10}$$

or more appropriately in calculus form

$$\frac{dN}{dt} = -\lambda N,$$

where λ is the proportionality constant known as the decay constant.

Solving this equation for N (using calculus) gives us

$$N(t) = N_0 e^{-\lambda t} \tag{4.11}$$

where N_0 is the number of radioactive nuclei at $t = 0$ and $e = 2.7182818284590452\ldots$, which is called the natural exponential.

Since $\frac{\Delta N}{\Delta t} \propto N$ then it must also decrease exponentially in time at the same rate:

$$\frac{\Delta N}{\Delta t} = \left(\frac{\Delta N}{\Delta t}\right)_0 e^{-\lambda t}. \tag{4.12}$$

The greater value of λ, the more radioactive a substance is considered to be.

A Note on e

The natural exponential, e, is interesting because of its mathematical properties. The function e^x is interesting (and useful) because it is the only (non-trivial) function whose slope is equal to its value at all points along the function. Mathematically,

$$\frac{d}{dx}e^x = e^x.$$

For example, $e^0 = 1$. The slope of e^x at $x = 0$ is also 1. This makes it especially useful in solving equations where the rate of change of a function is directly proportional to the function itself (such as with the radioactive decay equation).

Much like 10^x has an inverse function $\log_{10} x$, e^x's inverse function is $\log_e x$, also known as $\ln x$. You will see e^x and $\ln x$ as functions available to you on your calculator. In fact, e^x and $\ln x$ are much more useful functions than logs and exponents in base 10 when it comes to scientific applications.

Half-Life

Rather than discussing the decay constant, frequently we refer to a radioactive material's half-life ($T_{1/2}$) , that is the time, on average, when the number of radioactive nuclei is 1/2 the original. To determine the relationship between $T_{1/2}$ and λ we start with equation 4.11. Since $N(T_{1/2}) = \frac{1}{2}N_0$

$$\frac{1}{2}N_0 = N_0 e^{-\lambda T_{1/2}}$$

the N_0 divides out giving

$$\frac{1}{2} = e^{-\lambda T_{1/2}}.$$

Taking the natural logarithm of both sides we get

$$\ln\left(\frac{1}{2}\right) = \ln\left(e^{-\lambda T_{1/2}}\right).$$

Since $\ln \frac{1}{x} = -\ln x$ we get

$$\ln 2 = \lambda T_{1/2}.$$

So

$$T_{1/2} = \frac{\ln 2}{\lambda} \approx \frac{0.693}{\lambda} \tag{4.13}$$

Radioactive Dating

A certain percentage of all natural occurring materials is radioactive. For example, $1.1 \times 10^{-10}\%$ of all naturally occurring carbon is carbon-14. As long as an organism is alive, it replenishes its carbon, so the percentage of carbon-14 remains approximately constant. Once it dies, however, the carbon-14 percentage begins to drop as it undergoes β^--decay and changes to nitrogen-14.

By estimating the original amount of carbon-14 and the original activity $\left(\frac{\Delta N}{\Delta t}\right)_0$, based on the $1.1 \times 10^{-10}\%$, and by measuring the present activity $\frac{\Delta N}{\Delta t}$ we can estimate the age of the sample. The half life of carbon-14 is approximately 5730 years.

Radioactive Series

Frequently, an atom undergoes a series of radioactive decays to form a stable nucleus. This is the case with all nuclei larger than lead-208. Figure 4.1 shows an example of such a series of decays.

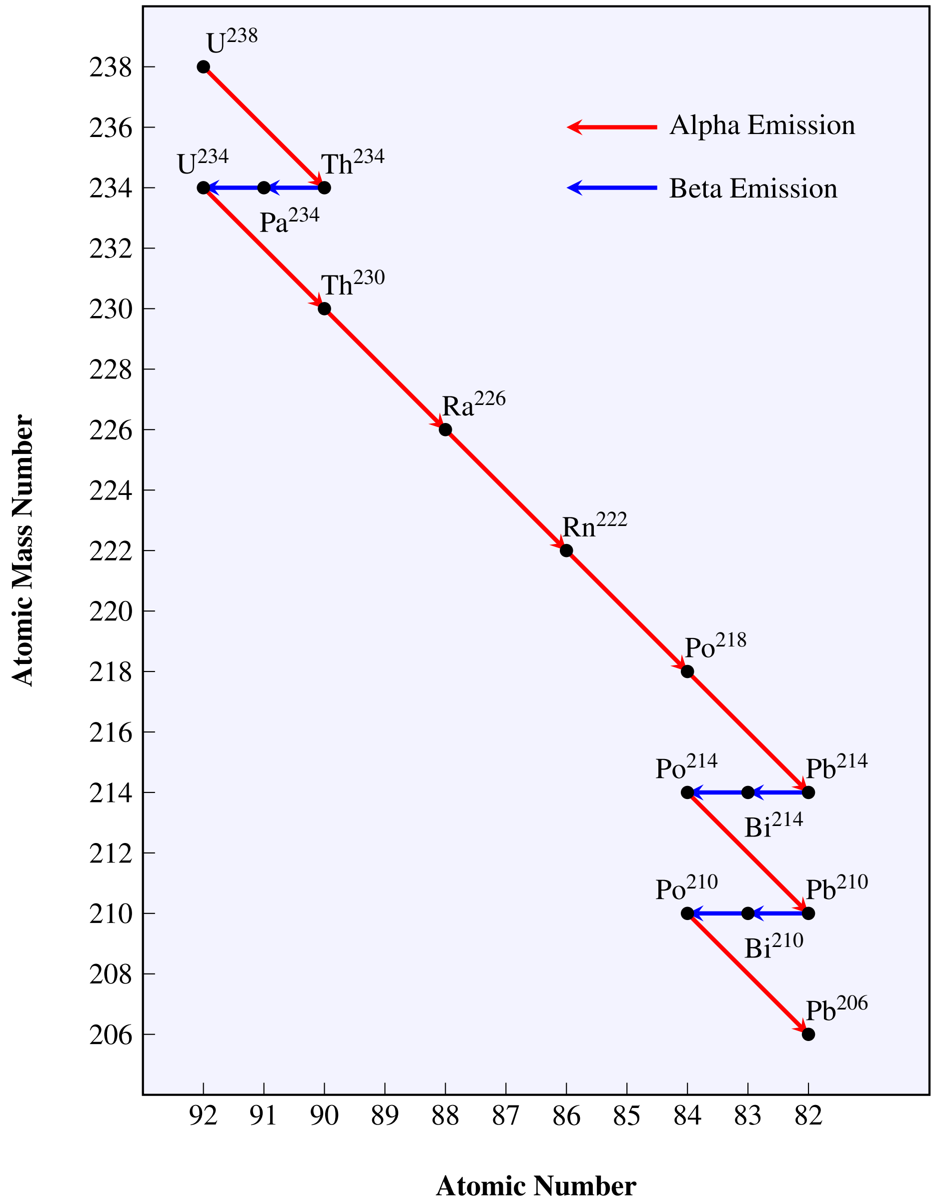

Figure 4.1: An example of a common decay series from Uranium-238 to Lead-206.