Chapter VI

Fluid Mechanics

Day 1 Fluid Mechanics - General Notes

Chapter 10

Density and Specific Gravity

Density is defined as the mass per unit volume, i.e.

$$\rho = \frac{m}{V}. \tag{6.1}$$

(There's nothing new here except that the standard symbol for density is the Greek letter "rho" instead of D). Specific gravity (SG) is the ratio of an object's density to that of water (i.e.,$SG = \frac{\rho}{\rho_{H_2O}}$). For a list of densities see Table 10-1 in your text.

Pressure

Pressure is the force per unit area applied perpendicularly to a surface, i.e. $P = \frac{F_\perp}{A}$. The pressure in a liquid of uniform density is dependent upon the depth of the liquid. The deeper we go, the more force is pushing down (more weight of fluid). So at a depth h, the pressure is given by

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{\rho Ahg}{A} = \rho gh. \tag{6.2}$$

Note that the volume of the fluid above any surface is equal to the area of the surface times the height of the fluid above.

Most pressure that we measure, however, is what is known as the ***gauge pressure***, which is the pressure differential from atmospheric pressure. In reality, we live in a fluid that is constantly applying pressure on us. The reason we aren't crushed under the weight of the force is that our cells have an internal pressure (pushing outward) approximately equal to that of the atmospheric pressure (a good reason to use pressurized cabins and suits in the upper atmosphere). ***Absolute pressure*** includes the atmospheric pressure and is given by

$$P = P_{atm} + P_{gauge}. \tag{6.3}$$

Pascal's Principle

Pascal said that any pressure applied in a confined fluid increases the pressure throughout the fluid by the same amount. This is really just another statement of Newton's 3rd law. The fluid will push back as hard at it is being pushed – since it's confined (can't go anywhere) it will push equally throughout the entire fluid.

Buoyancy

If we place an object under water (for example) it will appear to weigh less under the water than it will above. This is because of the variation of pressure at different depths. Consider figure 6.1. The top of the cylinder of height h and area A is a distance h_1 underneath the fluid. The force applied to the top of the cylinder by the water above it is $F_1 = \rho_F g h_1 A$, where ρ_F is the density of the fluid. Similarly, the force pushing up from the fluid is $F_2 = \rho_F g h_2 A$. The buoyant force (F_B) is equal to the difference of these forces, i.e.,

$$\begin{aligned} F_B &= F_2 - F_1 \\ &= \rho_F g A\,(h_2 - h_1) \\ &= \rho_F g A h \\ &= \rho_F g V \end{aligned}$$

(Note we have not included the force of gravity on the cylinder in this discussion - while it affects what happens to the cylinder, it does not affect the buoyant force.)

This leads us to Archimedes principle: *The buoyant force is equal to the weight of fluid displaced by the object.*

Please see examples 10-5 through 10-9 in your text book.

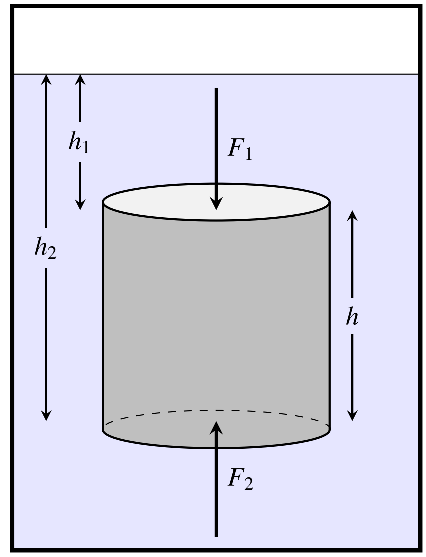

Figure 6.1: Buoyant force on cylinder under fluid

Fluid Flow Continuity

There are two types of fluid flow - ***laminar flow*** (or streamline flow) and ***turbulent flow***. In laminar flow, the fluid flows in layers (hence the name), which minimizes energy loss (although there is some loss due to the internal friction of these layers passing each other - this is called ***viscosity***). In turbulent flow, however, there are disturbances or eddies within the fluid, causing greater energy loss. We will be discussing primarily laminar flow.

We define the flow rate of a fluid as the ratio of the amount of mass passing a point to the period of time during which this occurs, i.e. Flow rate = $\frac{\Delta \mathrm{m}}{\Delta \mathrm{t}}$.

For laminar flow

$$\frac{\Delta m}{\Delta t} = \frac{\rho \Delta V}{\Delta t} = \frac{\rho A \Delta \ell}{\Delta t},$$

where A is the cross sectional area of the fluid and $\Delta\ell$ is the distance it travels in a time Δt.

But $\frac{\Delta m}{\Delta t} = v$, where v is the speed of the fluid flow, so

$$\frac{\Delta m}{\Delta t} = \rho A v.$$

If no fluid escapes, then at any two points along the flow of the fluid

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t},$$

so

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2.$$

But if the fluid is incompressible (which is approximately true for most liquids and some gases under certain circumstances) then $\rho_1 = \rho_2$,

$$\therefore A_1 v_1 = A_2 v_2$$

which is the equation of continuity.

See examples 10-10 and 10-11 in your text.

Bernoulli's Principle and Equation

Bernoulli's principle states that "where the velocity of a fluid is high, the pressure is low, and where the velocity of a fluid is low, the pressure is high." This, of course, is the explanation for how a plane's wings work, as well as why when you blow over a strip of paper it actually moves up rather than down.

Bernoulli's equation is based upon the work-energy principle (i.e. conservation of energy). The derivation can be found on p. 290 of your text book, but the end result is

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 = \text{constant.} \tag{6.4}$$

The first term is related to the internal energy of the fluid, the second related to the kinetic energy of the fluid and the third term is related to the gravitational potential of the fluid (note that if we multiple all these by the volume we get energy).

See example 10-12 in your text.